bending and shear design incl. verif. of serviceability (EC 2 (1.11), NA: Deutschland)
biaxial bending with/abs. axial force (4H-BETON version: 11/2007-41)

**rectangular section**

- b = 40.0 cm
- h = 40.0 cm
- edge distances of longitud. reinforcement
  - d₀ = 6.0 cm
  - d₁ = 6.0 cm
- material
  - C25/30
  - BSt 500 (A)
  - γs = 1.15, γc = 1.50
  - exposure class X0

**min./max. reinforcement**

<table>
<thead>
<tr>
<th>Nr</th>
<th>rank</th>
<th>min. Aₜ</th>
<th>max. Aₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.00</td>
<td>100.00</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

**design calculation values and minimum reinforcement areas (EC 2, 6.1)**

<table>
<thead>
<tr>
<th>γ</th>
<th>NEd</th>
<th>MEd</th>
<th>zEd</th>
<th>αₑ2u</th>
<th>αs2u</th>
<th>αₑ1u</th>
<th>αs1u</th>
<th>αₑkr</th>
<th>d₁</th>
<th>z₁</th>
<th>x₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>---</td>
<td>-1200.0</td>
<td>150.00</td>
<td>75.00</td>
<td>-3.50</td>
<td>2.61</td>
<td>1.54</td>
<td>2.43</td>
<td>120.27</td>
<td>46.5</td>
<td>29.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>27.36</td>
<td>0.00</td>
<td>-2.68</td>
<td>2.21</td>
<td>25.00</td>
<td>29.88</td>
<td>90.00</td>
<td>34.0</td>
<td>19.6</td>
<td>3.3</td>
</tr>
</tbody>
</table>

αₑ2u = concr. strain in state of failure (fibres 2), αs2u = reinforcement strain in state of failure (fibres 1), αₑkr = dir. angle of cross section principal strain, d₁ = static height, z₁ = lever arm of internal forces, x₁ = height of concr. compr. zone

8) minimum reinforcement acc. to 9.2.1.1

⇒ longitudinal reinforcement:  min Aₜ = 4.2/4.2/4.2/4.2 cm²

shear design calculation (EC 2, 6.2 + 6.3) - separated into VᵧEd + TEd and VzEd + TEd

minimum reinforcement acc. to 9.2.2.2(5), material quality as flexural reinforc.

z = 0.9 d (10.3.4(2), d in each direction), cₓ,d = 3.0 cm, D = compression reinforc.

angle of compr. strut: θ₁ = 0°, torsion: τₑfr = 10.0 cm

the minimum value of Vzdet is limited acc. to design code (Vzdet ≥ min Vzdet).

**design calculation of shear force (EC 2, 6.2)**

<table>
<thead>
<tr>
<th>VᵧEd</th>
<th>VzEd</th>
<th>zᵧ</th>
<th>VᵧEdt⁺</th>
<th>θᵧ</th>
<th>VzEdt⁺</th>
<th>θz</th>
<th>VzEdt⁻</th>
<th>θz</th>
<th>αₑ, νₜ</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.00</td>
<td>163.00</td>
<td>28.0</td>
<td>106.25</td>
<td>18.4</td>
<td>357.00</td>
<td>28.0</td>
<td>106.25</td>
<td>25.2</td>
<td>458.17</td>
<td>6.29</td>
</tr>
</tbody>
</table>

zᵧ: decisive inner lever arm. Vzdet: design value of shear resistance without shear reinforcement

θᵧ, θz: angle of compr. strut, Vzdet: design value of maximal shear resistance

**design calculation of torsion (EC 2, 6.3)**

<table>
<thead>
<tr>
<th>TEd</th>
<th>VᵧEdt⁺</th>
<th>θᵧ</th>
<th>VzEdt⁺</th>
<th>θz</th>
<th>TzEdt⁻</th>
<th>αₑ, νₜ</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00</td>
<td>25.83</td>
<td>18.4</td>
<td>49.08</td>
<td>25.2</td>
<td>51.54</td>
<td>0.30</td>
<td>2.30</td>
</tr>
</tbody>
</table>

stirrup reinforcement αₑ, νₜ per leg. longit. reinforc. for torsion Aₑ, νₜ uniformly distributed along the perimeter

**design calculation of shear force and torsion (EC 2, 6.3(4))**

1: (TEd / TzEdt⁻) + (VᵧEd / VzEdt⁻) = 0.05 < 1.0

(TEd / TzEdt⁻) + (VzEd / VzEdt⁻) = 0.14 < 1.0 ⇒ verification executed!
\[ \Rightarrow \text{shear reinforcement: } \min a_s,_{bV} = 6.29 \text{ cm}^2/m \]
\[ \text{torsion: } \min a_s,_{bT} = 0.30 \text{ cm}^2/m \] (1-shear)
\[ \Sigma \text{ (2-shear) } \min a_s,_{bU} = 6.90 \text{ cm}^2/m \]
\[ \text{torsion: } \min A_s,_{T} = 2.3 \text{ cm}^2 \] (uniformly distributed along the perimeter)

**crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)**

Cracking in bending restraint (self induced) factor for the concrete hardening process \( k_{c,t} = 1.00 \)

Axial force in the centre of gravity at formation of first crack \( N_{cr} = 0.00 \text{ kN} \)

Crack width \( w_c = 0.30 \text{ mm} \)

Crack forces and moments: \( N = -700.00 \text{ kN} \), \( M_y = 180.00 \text{ kNm} \), \( M_z = 65.00 \text{ kNm} \)

Reinforcement (initial state): \( A_s = 4.24/4/4.24/4.24 \text{ cm}^2 \)

<table>
<thead>
<tr>
<th>Nr</th>
<th>( k_c )</th>
<th>( k )</th>
<th>( A_s,_{min} )</th>
<th>( d_{eq} )</th>
<th>( \sigma_s )</th>
<th>( \Delta A_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16</td>
<td>0.40</td>
<td>0.74</td>
<td>1.37</td>
<td>33.2</td>
<td>166.8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>0.40</td>
<td>0.74</td>
<td>1.37</td>
<td>16.4</td>
<td>287.1</td>
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<td>16</td>
<td>0.40</td>
<td>0.74</td>
<td>1.37</td>
<td>16.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>0.40</td>
<td>0.74</td>
<td>1.37</td>
<td>16.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Cons.: tens. (str.) \( f_{ult} = 3.00 \text{ N/mm}^2 \)  conc. tens. strength (load) \( f_{ult} = f_{bm} = 2.66 \text{ N/mm}^2 \)

\( k_c \): coeff. - stress distribution, \( k \): coeff. - conc. tens. stress, \( A_s,_{min} \): min. rein. from restraint

\( d_{eq} \): selected bar diameter, \( d_{ex} \): existing bar diameter

\( \sigma_s \): tensile stress, \( \Delta A_s \): rein. increase from load and restraint

\[ \Rightarrow \text{incl. anti-crack reinforcement: } \min A_s = 6.5/6.5/6.5/6.5 \text{ cm}^2 \]

**Fatigue design (EC 2, 6.8.5 + 6.8.7(1))**

For steel: \( U_{sl} = \gamma_{ref} \sigma_{ref} \gamma_{dep,\sigma_{ref}} \Delta \sigma_{s,eq} \leq U_{sz} = \Delta \sigma_{sk} (N')/\gamma_{sf,\sigma_{ref}} = 152.17 \text{ N/mm}^2 \)

Damage equivalent stress range \( \Delta \sigma_{s,eq} = \sigma_{s,0} - \sigma_{s,eq} \)

Partial safety factors \( \gamma_{sf,\sigma_{ref}} = 1.00 \), \( \gamma_{dep,\sigma_{ref}} = 1.00 \), \( \gamma_{sf,\sigma_{ref}} = 1.15 \)

Allowable stress range \( \Delta \sigma_{sk} (N') = 175.0 \text{ N/mm}^2 \)

Shear force: \( \Delta \sigma_{sk} (N') = 107.0 \text{ N/mm}^2 \) \( \Rightarrow U_{sz} = \Delta \sigma_{sk} (N')/\gamma_{sf,\sigma_{ref}} = 93.04 \text{ N/mm}^2 \)

For conc.: \( U_{sl} = |\sigma_{cd,\max,eq} / f_{cd,\max,eq} + 0.43 \sqrt{1 - \sigma_{cd,\min,eq} / \sigma_{cd,\max,eq}} \| \leq 1.0 \)

Design value of compression strength \( f_{cd,\max,eq} = 15.00 \text{ N/mm}^2 \) at \( t_0 = 28 \text{ d} \)

Material safety \( \gamma_{sf,\sigma_{ref}} = 1.50 \)

Load: \( N_{sl} = -800.00 \text{ kN} \), \( M_{y1} = 90.00 \text{ kNm} \), \( M_{z1} = 75.00 \text{ kNm} \), \( M_{y2} = 0.00 \text{ kN} \), \( V_{y1} = 163.00 \text{ kNm} \), \( M_{z2} = 100.00 \text{ kNm} \), \( M_{y2} = 0.00 \text{ kN} \), \( V_{y2} = 0.00 \text{ kNm} \)

Reinforcement (initial state): \( A_s = 6.53/6.53/6.53/6.53 \text{ cm}^2 \), \( A_s,_{bV} = 6.29 \text{ cm}^2/m \)

**Fatigue design for steel:**

Concrete fatigue design:

\( \sigma_{cd,\min,eq} = 19.96 \text{ N/mm}^2 \)

\( \sigma_{cd,\max,eq} = 26.47 \text{ N/mm}^2 \)

\( U_{sl} = 1.98 < 1.00 \Rightarrow \text{verification not complied!} \)

Verification of compression strut for decisive \( V_{y1} \):

\( \sigma_{cd,\min,eq} = 0.00 \text{ N/mm}^2 \)

\( \sigma_{cd,\max,eq} = 3.62 \text{ N/mm}^2 \)

\( U_{cl} = 0.36 < 0.50 \Rightarrow \text{verification executed!} \)

Reinforcement (shear force):

\( \Delta \sigma_{sk,eq} = 93.04 - 0.00 = 93.04 \text{ N/mm}^2 = U_{sz} \)

\( \Rightarrow \Delta A_{sbV,\sigma} = 34.64 \text{ cm}^2/m \)

**Incl. fatigue reinforcement:**

\( \min A_s = 6.6/6.6/6.6/6.6 \text{ cm}^2 \)

\( \min A_s,_{bV} = 40.94 \text{ cm}^2/m \)

**Limitation of steel tension and concrete compression stresses (EC 2, 7.2)**

Permitted tensile stress of reinforcement \( \sigma_s = 0.80 \cdot f_{yk} = 400.0 \text{ N/mm}^2 \)

Permitted concrete compression stress \( \sigma_c = 0.60 \cdot f_{ck} = -15.0 \text{ N/mm}^2 \)

Stress forces and moments: \( N_s = -1200.00 \text{ kN} \), \( M_{y1} = 150.00 \text{ kNm} \), \( M_{z2} = 75.00 \text{ kNm} \)

Reinforcement (initial state): \( A_s = 6.88/6.88/6.88/6.88 \text{ cm}^2 \)

**Maximal reinforcement tensile stresses**

<table>
<thead>
<tr>
<th>Nr</th>
<th>( \sigma_{s0} )</th>
<th>( \sigma_s )</th>
<th>( \Delta A_{s0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
<td>0.0</td>
<td>32.52</td>
</tr>
<tr>
<td>2</td>
<td>104.3</td>
<td>28.8</td>
<td>32.52</td>
</tr>
<tr>
<td>3</td>
<td>-70.9</td>
<td>0.0</td>
<td>32.52</td>
</tr>
<tr>
<td>4</td>
<td>-165.1</td>
<td>0.0</td>
<td>32.52</td>
</tr>
</tbody>
</table>
minimal concrete compression stress

initial state:

\[ \sigma_{\text{c, initial}} = -25.8 \text{ N/mm}^2 \]

end state:

\[ \sigma_{\text{c, end}} = -15.0 \text{ N/mm}^2 > -15.0 \]

\[ \Rightarrow \text{incl. stress reinforcement: } \min A_3 = 39.4/39.4/39.4/39.4 \text{ cm}^2 (\text{max po !}) \]

fire protection acc. to EC2, Teil 1-2 (10.06)
mod. zone method (10 zones)
column flame application from left, right, top, bottom, fire duration 90 min
convective coeff. of thermal transfer \( \alpha = 25.0 \text{ W/mK} \), emissivity coeff. for concrete surface \( \varepsilon = 0.70 \)
normal dens. concr. with silicious aggr., moisture content 1.5%, upper limit of thermal conduct.
hot rolled reinforcing steel, density (reinforced concrete) \( \rho = 2300 \text{ kg/m}^3 \)
assumption for the design calculation: concrete temperature of the coldest cross-section point (point M) assumption for the design calculation: no inner stresses to be taken into account
assumption for the design calculation: stress-strain relation form acc. to EC 2 (fire case)

simplified method for transient heat transport
finite volume method with explicit time integration taking into account non-linear material and boundary conditions
temperature profile (90 min, rotated 0°):
\[ \text{dx} = 0.50 \text{ cm} \quad \text{dy} = 0.50 \text{ cm} \ (6861 \text{ cell nodes}), \min \Delta t = 0.056 \text{ min} \]

horizontal section through point M:

![Diagram showing temperature profile and horizontal section through point M](image)

temperatures for 10 zones with related reduction factors:
\[ \Theta_1 = 788.4^\circ \text{C}, \quad k_{c1} = 0.167 \quad \Theta_2 = 522.2^\circ \text{C}, \quad k_{c2} = 0.567 \quad \Theta_3 = 357.4^\circ \text{C}, \quad k_{c3} = 0.793 \]
\[ \Theta_4 = 249.3^\circ \text{C}, \quad k_{c4} = 0.901 \quad \Theta_5 = 175.7^\circ \text{C}, \quad k_{c5} = 0.962 \quad \Theta_6 = 127.2^\circ \text{C}, \quad k_{c6} = 0.986 \]
\[ \Theta_7 = 98.8^\circ \text{C}, \quad k_{c7} = 1.000 \quad \Theta_8 = 81.8^\circ \text{C}, \quad k_{c8} = 1.000 \quad \Theta_9 = 71.4^\circ \text{C}, \quad k_{c9} = 1.000 \]
\[ \Theta_{10} = 66.4^\circ \text{C}, \quad k_{c10} = 1.000 \]

mean reduction factor (related temperature):
\[ k_{cm} = 0.838 \quad (k_{cm} = 312.4^\circ \text{C}) \]

temperature in point M with associated reduction factor: \( \Theta_{M} = 65.5^\circ \text{C}, \quad k_{cM} = 1.000 \)
static ineffective concrete boundary zone: \( a_{z1} = 4.21 \text{ cm} \quad a_{z2} = 4.21 \text{ cm} \quad a_{z3} = 4.21 \text{ cm} \quad a_{z4} = 4.21 \text{ cm} \)
concrete temperature (design calculation) with associated reduction factor: \( \Theta_{c} = 65.5^\circ \text{C}, \quad k_{c} = 1.000 \)
reinforcement temperatures:
\[ k_{s1} = 466.8^\circ \text{C} \quad k_{s2} = 466.8^\circ \text{C} \quad k_{s3} = 466.8^\circ \text{C} \quad k_{s4} = 466.8^\circ \text{C} \]
associated reduction factors:
\[ k_{s1} = 0.853 \quad k_{s2} = 0.853 \quad k_{s3} = 0.853 \quad k_{s4} = 0.853 \]
\[ k_{sp1} = 0.380 \quad k_{sp2} = 0.380 \quad k_{sp3} = 0.380 \quad k_{sp4} = 0.380 \]
\[ k_{es1} = 0.633 \quad k_{es2} = 0.633 \quad k_{es3} = 0.633 \quad k_{es4} = 0.633 \]

fire protection for \( \gamma_c = \gamma_s = 1 \) (parameters of stress-strain relation acc. to 3.2)
reduced cross-section: \( b = 31.58 \text{ cm} \quad h = 31.58 \text{ cm} \)
design calculation values:
\[ N_{Ed, f1} = -1200.00 \text{ kN} \quad M_{Ed, f1} = 150.00 \text{ kNm} \quad M_{Ed, f1} = 75.00 \text{ kNm} \]
material properties:
concrete: \( \Theta = 65^\circ \text{C}; \quad f_{c, \Theta} = 25.0 \text{ N/mm}^2 \quad (E_{c, \Theta} = 31475.8 \text{ N/mm}^2) \)
reinforcement 1: \( \Theta = 467^\circ \text{C}; \quad f_{sp, \Theta} = 190.0 \text{ N/mm}^2 \quad f_{sy, \Theta} = 426.5 \text{ N/mm}^2 \quad E_{s, \Theta} = 126640.3 \text{ N/mm}^2 \)
reinforcement 2: \( \Theta = 467^\circ \text{C}; \quad f_{sp, \Theta} = 190.0 \text{ N/mm}^2 \quad f_{sy, \Theta} = 426.5 \text{ N/mm}^2 \quad E_{s, \Theta} = 126640.3 \text{ N/mm}^2 \)
\[\varepsilon_{SP,0} = 1.50\% \quad \varepsilon_{SY,0} = 20.00\% \quad \varepsilon_{ST,0} = \varepsilon_{SU,0} = 50.00\% \quad \varepsilon_{YV,0} = 0.00\%\]

**reinf. gr. 3** \(\theta = 46^\circ C\): \(f_{SP,0} = 190.0 \text{ N/mm}^2\) \(f_{SY,0} = 426.5 \text{ N/mm}^2\) \(E_{s,0} = 126640.3 \text{ N/mm}^2\)

\[\varepsilon_{SP,0} = 1.50\% \quad \varepsilon_{SY,0} = 20.00\% \quad \varepsilon_{ST,0} = \varepsilon_{SU,0} = 50.00\% \quad \varepsilon_{YV,0} = 0.00\%\]

**reinf. gr. 4** \(\theta = 46^\circ C\): \(f_{SP,0} = 190.0 \text{ N/mm}^2\) \(f_{SY,0} = 426.5 \text{ N/mm}^2\) \(E_{s,0} = 126640.3 \text{ N/mm}^2\)

\[\varepsilon_{SP,0} = 1.50\% \quad \varepsilon_{SY,0} = 20.00\% \quad \varepsilon_{ST,0} = \varepsilon_{SU,0} = 50.00\% \quad \varepsilon_{YV,0} = 0.00\%\]

\[\Rightarrow \text{fire reinforcement:} \quad \text{min } A_{S,v} = 9.67/9.67/9.67/9.67 \text{ cm}^2\]

total reinforcement: \(\text{total } A_S = 39.4/39.4/39.4/39.4 \text{ cm}^2 \) (max \(\rho_D\) !)

\[\text{total } A_{S,ov} = 40.94 \text{ cm}^2/m\]

\[\text{total } A_{S,ovT} = 0.30 \text{ cm}^2/m, \quad A_{S,T} = 2.3 \text{ cm}^2\]

degree of utilization: \(\rho = 0.29\)

**selected:**
- longitudinal, \(E_1\): 1 \(\varnothing 25 = 4.9 \text{ cm}^2 < 39.4 \text{ cm}^2\)
- \(E_2\): 1 \(\varnothing 25 = 4.9 \text{ cm}^2 < 39.4 \text{ cm}^2\)
- \(E_3\): 1 \(\varnothing 25 = 4.9 \text{ cm}^2 < 39.4 \text{ cm}^2\)
- \(E_4\): 1 \(\varnothing 25 = 4.9 \text{ cm}^2 < 39.4 \text{ cm}^2\)

stirrups, 2-shear: \(\varnothing 8 / 30 \text{ cm} = 3.35 \text{ cm}^2/m < 41.54 \text{ cm}^2/m\)

cross-section data

gross area of concrete: \(A_C = 16.0 \text{ dm}^2\)

second moment of area: \(I_{CYS} = 21.3 \text{ dm}^4\), \(I_{CZS} = 21.3 \text{ dm}^4\)

centroid coordinates (from centre of upper edge): \(y_s = 0.0 \text{ cm}, \quad z_s = 20.0 \text{ cm}\)

total area of longitudinal reinforcement: \(\Sigma (\text{min } A_S) = 157.6 \text{ cm}^2 \Rightarrow \rho_S = 9.85\% > 8.00\%\)

**material properties for design calculation**

concrete \(f_{ck} \quad \alpha \quad \varepsilon_{C2} \quad \varepsilon_{C2u} \quad n_{C} \quad E_{cm} \quad f_{cm} \quad f_{ctm}\)

<table>
<thead>
<tr>
<th></th>
<th>MN/m²</th>
<th>-</th>
<th>%</th>
<th>%</th>
<th>MN/m²</th>
<th>MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>C25/30</td>
<td>25.0</td>
<td>0.85</td>
<td>-2.00</td>
<td>-3.50</td>
<td>2.00</td>
<td>31475.8</td>
</tr>
</tbody>
</table>

design value of compression strength \(f_{cd} = \varepsilon_{C2} f_{ck} / \gamma_C\)

strain at reaching the maximum strength \(\varepsilon_{C2}\), ult. compr. strain \(\varepsilon_{C2u}\)

concr. comp. stress \(\varepsilon_C = f_{cd} (1-(1-0.8\gamma_C))\) for \(\varepsilon_{C2} < \varepsilon_C < \varepsilon_{C2u}\)

modulus of elasticity \(E_{cm}\), mean value of axial tensile strength \(f_{cm}\)

**reinforcement:** \(f_{yk} \quad f_{tk} \quad \varepsilon_{SU} \quad E_s\)

<table>
<thead>
<tr>
<th></th>
<th>MN/m²</th>
<th>MN/m²</th>
<th>%</th>
<th>MN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>BST 500 (A)</td>
<td>500.0</td>
<td>525.0</td>
<td>25.0</td>
<td>200000.0</td>
</tr>
</tbody>
</table>

design yield strength \(f_{yd} = f_{yk} / \gamma_y\)

design tensile strength \(f_{tu} = f_{tk} / \gamma_y\)

ult. tensile strain \(\varepsilon_{SU}\), modulus of elasticity \(E_s\)

**symbols:** positive result values marked with -1.0 or **** in tables refer to incorrect resp. not computable conditions!