bending and shear design incl. verif. of serviceability (EC 2 (1.11), NA: Deutschland)
biaxial bending with/abs. axial force (4H-BETON version: 11/2007-41)

T-beam-cross section

- h = 75.0 cm  h₀ = 16.0 cm
- b₀ = 145.0 cm  bₜ = 25.0 cm

edge distances of longitudinal reinforcement
- d₀ = 6.0 cm  dₜ = 6.0 cm
- d₁ = 6.0 cm  dₚ = 6.0 cm

Material:
C25/30
Bst 500 (A)
γₛ = 1.15, γₜ = 1.50
Exposure class X0

Minimum/maximum reinforcement
- min $Aₚ$ (9.2.1.1, 9.5.2), max $ρ₀ = 8.00$% for reinforcement groups

- $N_r$: rank  $Aₚ$: min $Aₚ$ max $Aₚ$
- cm² cm²

<table>
<thead>
<tr>
<th>rank</th>
<th>$N_r$</th>
<th>min $Aₚ$</th>
<th>max $Aₚ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>20.00</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>25.00</td>
<td>100.00</td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td>15.00</td>
<td>100.00</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>30.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

min $Aₚ$: initial reinforcement per group
max $Aₚ$: highest reinforcement amount per group

Ranking variable by load cases

Stirrups: $min\ a_{s0} = 40.00$ cm²/m

Verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (Figure 3.3)
with $f_{ck} = \frac{f_{c/k}}{\gamma} = 14.2$ MN/m² and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $fy = f_y = 434.8$ MN/m²
and $fₚ = f_k / \gammaₚ = 456.5$ MN/m²

Verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (Figure 3.2)
with $f_{ck} = f_{cm} = 33.0$ MN/m² and reinforcement stress-strain relation acc. to 3.2.7 (Figure 3.8) with $fy = f_y = 525.0$ MN/m² and $\gamma_k = 25$%.

Design Calculation Values and Minimum Reinforcement Areas (EC 2, 6.1)

$$
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\gamma & N_{Ed} & M_{Ed} & M_{Ed} & f_{c2} & f_{s2} & f_{s1} & f_{a} & \omega & d & z & x \\
\hline
1 & 90.00 & 100.00 & 300.00 & 3.50 & 0.52 & 5.92 & 13.65 & 209.34 & 73.7 & 65.4 & 27.4 \\
\hline
\end{array}
$$

- As₂: compression strain in state of failure (fibre 2), $f_{s2}$: reinforcement strain in state of failure (fibre 1),
- $\omega$: dir. cross sec. principal strain, $d$: static height, $z$: lever arm of internal forces, $x$: height of compression compr. zone

<table>
<thead>
<tr>
<th>$A_{sb1}$</th>
<th>$A_{sb2}$</th>
<th>$A_{sb3}$</th>
<th>$A_{sb4}$</th>
<th>note</th>
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<tbody>
<tr>
<td>4.14 cm²</td>
<td>4.14 cm²</td>
<td>4.14 cm²</td>
<td>4.14 cm²</td>
<td>12)</td>
</tr>
<tr>
<td>0.22 cm²</td>
<td>0.22 cm²</td>
<td>0.22 cm²</td>
<td>0.22 cm²</td>
<td>8) 12)</td>
</tr>
</tbody>
</table>

8) Minimum reinforcement acc. to 9.2.1.1 12) Uniaxial design calculation uneconomic

$$\Rightarrow$$ Longitudinal reinforcement: $min\ Aₚ = 20.0 / 25.0 / 15.0 / 30.0$ cm²

Shear Design Calculation (EC 2, 6.2 + 6.3) - separated into $V_{yd} + T_{Ed}$ and $V_{yd} + T_{Ed}$

Minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinforcement.
$z = 0.9 (10.3.4(2), d$ in each direction), $c_v = 3.0$ cm, $D =$ compression reinforcement
angle of compr. strut $\theta_{ges} = 0^\circ$, torsion: $\theta_{terr} = A_s / A_c > 2 \cdot min(d_{c}, d_{u})$
the minimum value of $V_{det}$ is limited acc. to design code ($V_{det} \geq min\ V_{det}$).
only web design; compression of compression/tension boom to be designed separately

Design Calculation of Shear Force (EC 2, 6.2)

$$
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
V_{yd} & V_{yd} & z_y & V_{ydmax} & z_z & V_{zd} & 0_z & V_{zdmax} & a_{s, h} & \text{cm²/m} \\
\hline
100.00 & 45.00 & 13.0 & 79.43 & 33.8 & 479.16 & 62.1 & 66.63 & 30.2 & 716.75 & 11.86 \\
\hline
\end{array}
$$

8: decisive inner lever arm, $V_{Ed, d}$: design value of shear resistance without shear reinforcement
$\theta$: angle of compr. strut, $V_{Ed, d}$: design value of maximal shear resistance

Design Calculation of Torsion (EC 2, 6.3)

$$
\begin{array}{|c|c|c|c|}
\hline
T_{Ed} & V_{EdT} & T_{Edmax} & a_{s, h} & As & T & \text{note} \\
\hline
35.00 & 39.17 & 33.8 & 128.88 & 30.2 & 66.14 & 2.63 & 10.97 \\
\hline
\end{array}
$$

Stirrup reinforcement $a_{s, h} / T$ per leg, longitudinal reinforcing $As_T$ uniformly distributed along the perimeter
design calculation of shear force and torsion (EC 2, 6.3(4))

1: \( \frac{T_{Ed}}{T_{Rm,ax}} + \frac{V_{Sy,Ed}}{V_{Rm,ax}} = 0.32 < 1.0 \)
\( \frac{T_{Ed}}{T_{Rm,ax}} + \frac{V_{Vz,Ed}}{V_{Rm,ax}} = 0.32 < 1.0 \) ⇒ verification executed!

⇒ shear reinforcement: \( \min a_{s,bo} = 40.00 \text{ cm}^2/\text{m} \)

\( \text{incl. initial reinft.} \)

\[ \begin{align*}
\text{torsiion: } & \min a_{s,bo} = 2.63 \text{ cm}^2/\text{m} \quad (1\text{-shear}) \\
\Sigma (2\text{-shear}): & \min a_{s,bo} = 45.26 \text{ cm}^2/\text{m} \\
\text{torsiion: } & \min A_{s,t} = 11.0 \text{ cm}^2 \\
& \text{(uniformly distributed along the perimeter)}
\end{align*} \]

crack control (EC 2, 7.3.2 minimum reinforcement, 7.3.3 without direct calculation) cracking in bending restraint (self induced)

factor for the concrete hardening process \( k_{c,t} = 1.00 \)

axial force in the centre of gravity at formation of first crack \( N_{cr} = 0.00 \text{ kN} \)

crack width \( w_k = 0.30 \text{ mm} \)

crack forces and moments: \( N = 75.00 \text{ kN} \)
\( M_{y,y} = 100.00 \text{ kNm} \)
\( M_{z,y} = 165.00 \text{ kNm} \)

reinforcement (initial state): \( A_s = 20.00/25.00/15.00/30.00 \text{ cm}^2 \)

<table>
<thead>
<tr>
<th>Nr</th>
<th>( d_s )</th>
<th>( k_c )</th>
<th>( k )</th>
<th>( A_{s,min} )</th>
<th>( d_{ef} )</th>
<th>( \sigma_s )</th>
<th>( A_{s,t} )</th>
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<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.40</td>
<td>0.80</td>
<td>1.79</td>
<td>60.0</td>
<td>32.2</td>
<td>0.00</td>
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<td>2</td>
<td>20</td>
<td>0.40</td>
<td>0.80</td>
<td>1.79</td>
<td>60.0</td>
<td>49.2</td>
<td>0.00</td>
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<tr>
<td>3</td>
<td>20</td>
<td>0.40</td>
<td>0.80</td>
<td>1.79</td>
<td>60.0</td>
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<td>20</td>
<td>0.40</td>
<td>0.80</td>
<td>1.79</td>
<td>60.0</td>
<td>31.3</td>
<td>0.00</td>
</tr>
</tbody>
</table>

conc. tens. str. (restr.): \( f_{ef} = 2.56 \text{ N/mm}^2 \)
conc. tens. strength (load): \( f_{ef} = 2.56 \text{ N/mm}^2 \)

\( k_c \): coeff. - stress distribution, \( k \): coeff. - concr. tens. stress, \( A_{s,min} \): min. reinft. from restraint

\( d_s \): selected bar diameter, \( d_{ef} \): existing bar diameter

\( \sigma_s \): steel tensile stress, \( A_{s,t} \): reinft. increase from load and restraint

⇒ no additional anti-crack reinforcement!

fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: \( U_{s1} = \gamma_{sf,t} \gamma_{ed,fat} \Delta s_{equ} \leq U_{s2} = \Delta s_{sk} (N^*)/\gamma_{sf,t} \)

allowable stress range \( \Delta s_{sk} = \sigma_{s,0} - \sigma_{s,U} \)

partial safety factors \( \gamma_{sf,t} = 1.00 \), \( \gamma_{ed,fat} = 1.00 \), \( \gamma_{sf,t} = 1.15 \)

load: \( N_{s1} = 25.00 \text{ kN} \)
\( M_{y,y} = 55.00 \text{ kNm} \)
\( M_{z,y} = 135.00 \text{ kNm} \)
\( M_{y,z} = 100.00 \text{ kN} \)
\( M_{z,y} = 80.00 \text{ kNm} \)
\( M_{y,z} = 125.00 \text{ kNm} \)

reinforcement (initial state): \( A_s = 20.00/25.00/15.00/30.00 \text{ cm}^2 \)

\( A_{s,bo} = 40.00 \text{ cm}^2/\text{m} \)

fatigue design for steel:

<table>
<thead>
<tr>
<th>Nr</th>
<th>( \Delta s_{equ} )</th>
<th>( \Delta s_{sk} )</th>
<th>( U_{s1} )</th>
<th>( U_{s2} )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>25.81</td>
<td>14.54</td>
<td>11.27</td>
<td>22.21</td>
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<tr>
<td>2</td>
<td>31.96</td>
<td>29.10</td>
<td>2.86</td>
<td>8.00</td>
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<tr>
<td>3</td>
<td>43.16</td>
<td>13.70</td>
<td>29.46</td>
<td>0.00</td>
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</tbody>
</table>

fatigue design for concrete:

<table>
<thead>
<tr>
<th>Nr</th>
<th>( \Delta s_{equ} )</th>
<th>( \Delta s_{sk} )</th>
<th>( U_{s1} )</th>
<th>( U_{s2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.21</td>
<td>25.81</td>
<td>14.54</td>
<td>12.27</td>
</tr>
<tr>
<td>2</td>
<td>27.95</td>
<td>31.96</td>
<td>29.10</td>
<td>2.86</td>
</tr>
<tr>
<td>3</td>
<td>39.05</td>
<td>43.16</td>
<td>13.70</td>
<td>29.46</td>
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<tr>
<td>4</td>
<td>28.59</td>
<td>28.59</td>
<td>7.55</td>
<td>21.04</td>
</tr>
</tbody>
</table>

⇒ no additional fatigue reinforcement!

reinforcement (shear forces):

\( \Delta s_{equ} = 111.03 - 88.82 = 22.21 \text{ N/mm}^2 \)
\( U_{s1} = 22.21 < U_{s2} = 93.04 \)
\( \Delta s_{equ} = 29.05 - 18.59 = 10.46 \text{ N/mm}^2 \)
\( U_{s1} = 10.46 < U_{s2} = 93.04 \)

⇒ no additional fatigue reinforcement!
limitation of steel tension and concrete compression stresses (EC 2, 7.2)
permitted tensile stress of rein. $\sigma_s = 0.80 \cdot f_{yk} = 400.0 \text{ N/mm}^2$
permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck} = -15.0 \text{ N/mm}^2$
stress forces and moments: $N_o = 0.00 \text{ kN}$, $M_y = 100.00 \text{ kNm}$, $M_{yz} = 300.00 \text{ kNm}$
reinforcement (initial state): $A_s = 20.00/25.00/15.00/30.00 \text{ cm}^2$

<table>
<thead>
<tr>
<th>Nr</th>
<th>$\sigma_s$</th>
<th>$\sigma_c$</th>
<th>$\Delta \sigma_{as}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.0</td>
<td>19.5</td>
<td>1.13</td>
</tr>
<tr>
<td>2</td>
<td>54.0</td>
<td>51.5</td>
<td>1.33</td>
</tr>
<tr>
<td>3</td>
<td>95.9</td>
<td>91.6</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>62.9</td>
<td>59.5</td>
<td>1.54</td>
</tr>
</tbody>
</table>

$\sigma_s$: initial state, $\sigma_c$: end state
$\Delta \sigma_{as}$: reinforcement increase from steel and concrete design

$\Rightarrow$ incl. stress reinforcement: $\min A_s = 21.1/26.3/16.0/31.5 \text{ cm}^2$

total reinforcement: $A_s = 21.1/26.3/16.0/31.5 \text{ cm}^2$
total $a_{s,b} = 40.00 \text{ cm}^2/m$
total $a_{s,\text{bar}} = 2.63 \text{ cm}^2/m$, $A_{s,T} = 11.0 \text{ cm}^2$

degree of utilization: $U = 0.56$

additional reinforcement: $\Delta A_s = 1.1/1.3/1.0/1.5 \text{ cm}^2$

selected: longitudinal, El: 1 $\varnothing 25 = 4.9 \text{ cm}^2 < 21.1 \text{ cm}^2$
E2: 1 $\varnothing 25 = 4.9 \text{ cm}^2 < 26.3 \text{ cm}^2$
E3: 1 $\varnothing 25 = 4.9 \text{ cm}^2 < 16.0 \text{ cm}^2$
E4: 1 $\varnothing 25 = 4.9 \text{ cm}^2 < 31.5 \text{ cm}^2$

stirrups, 2-shear: $\varnothing 8 / 15 \text{ cm} = 6.70 \text{ cm}^2/m < 45.26 \text{ cm}^2/m$

cross-section data
gross area of concrete: $A_c = 37.9 \text{ dm}^2$ second moment of area: $I_{gs} = 174.5 \text{ dm}^4$, $I_{cgs} = 414.2 \text{ dm}^4$
centroid coordinates (from centre of upper edge): $y_s = 0.0 \text{ cm}$, $z_s = 22.6 \text{ cm}$
total area of longitudinal reinforcement: $\Sigma(a_{s}) = 95.0 \text{ cm}^2 \Rightarrow p_s = 2.50% < 8.00%$

material properties for design calculation

<table>
<thead>
<tr>
<th>concrete</th>
<th>$f_{ck}$</th>
<th>$\alpha$</th>
<th>$e_{c2}$</th>
<th>$e_{c2u}$</th>
<th>$\phi_c$</th>
<th>$E_m$</th>
<th>$f_{ctm}$</th>
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<tbody>
<tr>
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<td>%</td>
<td>%</td>
<td>MN/m$^2$</td>
<td>MN/m$^2$</td>
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<tr>
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<td>-3.50</td>
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<td>31475.8</td>
<td>2.565</td>
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</table>

reinforcement

<table>
<thead>
<tr>
<th>$f_{yk}$</th>
<th>$f_{tk}$</th>
<th>$e_{su}$</th>
<th>$E_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN/m$^2$</td>
<td>MN/m$^2$</td>
<td>%</td>
<td>MN/m$^2$</td>
</tr>
<tr>
<td>BSt 500 (A)</td>
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<td>525.0</td>
<td>25.00</td>
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</table>

<table>
<thead>
<tr>
<th>$f_{sd}$</th>
<th>$f_{ck}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN/m$^2$</td>
<td>MN/m$^2$</td>
</tr>
<tr>
<td>design value of compression strength $f_{sd} = \phi_c \cdot f_{ck} / \gamma_c$</td>
<td></td>
</tr>
</tbody>
</table>

strain at reaching the maximum strength $e_{c2} = f_{ck} / E_m$,
concr. comp. stress $\sigma_c = f_{cd} (1 - e_c / \varepsilon_{c2})$ for $\varepsilon_{c2} < e_{c2u}$ and $\sigma_c = f_{ct} / \varepsilon_{c2}u$ for $e_{c2} > e_{c2u}$
modulus of elasticity $E_{cm}$, mean value of axial tensile strength $f_{ct}$

<table>
<thead>
<tr>
<th>$f_{yd}$</th>
<th>$f_{tk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MN/m$^2$</td>
<td>MN/m$^2$</td>
</tr>
<tr>
<td>design yield strength $f_{yd} = f_{tk} / \gamma_y$</td>
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</tr>
<tr>
<td>design tensile strength $f_{yd} = f_{tk} / \gamma_y$</td>
<td></td>
</tr>
<tr>
<td>ult. tensile strain $e_{su}$, modulus of elasticity $E_s$</td>
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</tbody>
</table>