POS. 19: T-BEAM (REINFORCED CONCRETE 1-ACHS.)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)

uniaxial bending with/without axial force (4H-BETON: version 11/2007-4)

detailing of reinforcement
preferably in tension face ($\varepsilon_{\text{slu}} = 25.00\%$)

min./max. reinforcement
$\min A_s$ (9.2.1.1, 9.5.2), $\max \rho_0 = 8.00\%$

initial reinforcement
$A_{s00} = 0.00 \text{ cm}^2$, $A_{su} = 0.00 \text{ cm}^2$

$A_{s050} = 0.00 \text{ cm}^2/m$

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)

with $f_{cd} = \frac{f_{ck}}{\gamma_c} = 14.2 \text{ MN/m}^2$ and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $f_y = f_{yk}/\gamma_y = 434.8 \text{ MN/m}^2$

and $f_t = f_{tk}/\gamma_t = 456.5 \text{ MN/m}^2$!

verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)

with $f_c = f_{ck} = 33.0 \text{ MN/m}^2$ and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $f_y = f_{yk}$, $f_t = 525.0 \text{ MN/m}^2$ and $\gamma_y = 25\%$!

design calculation values and minimum reinforcement areas (EC 2, 6.1)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$N_{\text{Ed}}$</th>
<th>$M_{\text{Ed}}$</th>
<th>$\varepsilon_{\text{clu}}$</th>
<th>$\varepsilon_{\text{slu}}$</th>
<th>$\varepsilon_{\text{clu}}$</th>
<th>$\varepsilon_{\text{clu}}$</th>
<th>$\varepsilon_{\text{clu}}$</th>
<th>$\zeta$</th>
<th>$\zeta$</th>
<th>$d$</th>
<th>$A_{s0}$</th>
<th>$A_{su}$</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>---</td>
<td>$-125.00$</td>
<td>$500.00$</td>
<td>$-2.29$</td>
<td>$-0.78$</td>
<td>$25.00$</td>
<td>$27.63$</td>
<td>$0.08$</td>
<td>$0.97$</td>
<td>$68.4$</td>
<td>---</td>
<td>$15.61$</td>
<td></td>
</tr>
</tbody>
</table>

with $\varepsilon_{\text{clu}} = -3.50\%$, concr. strain in state of failure (fiber 2), $\varepsilon_{\text{slu}} = 25.00\%$, reinforcement strain in state of failure (fiber 1)

$X = \zeta d$: height of concr. compr. zone, $z - \zeta d$: lever arm of internal forces, $d = h - d$: effective depth

9) minimum reinforcement acc. to 9.2.1.1

$\Rightarrow$ longitudinal reinforcement: $\min A_{s0} = 0.0 \text{ cm}^2$, $\min A_{su} = 15.6 \text{ cm}^2$

shear design calculation (EC 2, 6.2 + 6.3)

minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf,

$z = 0.9 d$ (6.2.3(1)), $c_v = 3.0 \text{ cm}$, $D =$ compression reinf.

angle of reinforcement $\alpha = 90^0, \gamma_a = 90^0$

the minimum value of $V_{\text{mdc}}$ is limited acc. to design code ($V_{\text{mdc}} \geq \min V_{\text{mdc}}$).

only web design; connection of compression/tension boom to be designed separately.

design calculation of shear force (EC 2, 6.2)

<table>
<thead>
<tr>
<th>$V_{\text{Ed}}$</th>
<th>$p_1$</th>
<th>$z$</th>
<th>$V_{\text{mdc}}$</th>
<th>$\theta$</th>
<th>$\cot \theta$</th>
<th>$V_{\text{max}}$</th>
<th>$A_B$</th>
<th>$a_1$</th>
<th>$a_{s,bw}$</th>
<th>$\text{ note}$</th>
</tr>
</thead>
</table>
| 1 | $50.00$ | $0.91$ | $61.6$ | $88.41$ | $18.4$ | $3.00$ | $490.56$ | $1$ | $92.3$ | $2.05$ | $\text{minimum reinforcement}$

with $p_1$ ratio of longit. reinf. related to static height, $z$: decisive inner lever arm

$V_{\text{mdc}}$: design value of shear resistance without shear reinforcement, $\theta$: angle of compr. strut,

$V_{\text{mdc}}$: design value of maximal shear resistance, $a_1$: shift rule

$A_B$: range of utilization see NA/DE

$\Rightarrow$ shear reinforcement: $\min a_{s,bw} = 2.05 \text{ cm}^2/m$

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)

cracking in bending restraint (intrinsically imposed)

minimum reinforcement:

cof. - stress distribution $k_c = 0.55 / 0.26$

cof. - selfequil. stresses $k = 0.80$

concr. tens. str. (restr.) $f_{ct,eff} = 2.56 \text{ N/mm}^2$

tension zones $A_{ct,0} = 6.2 \text{ dm}^2$, $A_{ct,1} = 12.5 \text{ dm}^2$

$A_{st,0} = 1.6 \text{ cm}^2$, $A_{st,1} = 6.6 \text{ cm}^2$

$\text{crack control:}$

concr. tens. strength (load) $f_{ct,eff} = f_{ct} = 2.56 \text{ N/mm}^2$

effective slab width $b_{eff} = 36.4 \text{ cm}$

$\sigma_0 = 0.0 \text{ N/mm}^2$, $\sigma_{su} = 284.9 \text{ N/mm}^2$
fatigue design (EC 2, 6.8.5 + 6.8.7(1))
for steel: \( U_{si} = \gamma_f,_{fat} \gamma_{cd,_{fat}} \Delta \sigma_{s,_{equ}} \leq U_{s2} = \Delta \sigma_{sk} (N^\ast) / \gamma_{s,_{fat}} = 152.17 \text{ N/mm}^2 \)
damage equivalent stress range \( \Delta \sigma_{s,_{equ}} = \sigma_{0,0} - \sigma_{U,_{s}} \)
partial safety factors \( \gamma_f,_{fat} = 1.00, \gamma_{cd,_{fat}} = 1.00, \gamma_{s,_{fat}} = 1.15 \)
allowable stress range \( \Delta \sigma_{sk} (N^\ast) = 175.0 \text{ N/mm}^2 \)
shear force: \( \Delta \sigma_{sk} (N^\ast) = 107.0 \text{ N/mm}^2 \Rightarrow U_{s2} = \Delta \sigma_{sk} (N^\ast) / \gamma_{s,_{fat}} = 93.04 \text{ N/mm}^2 \)
for conc.: \( U_{ci} = \left[ f_{cd,_{max,_{equ}}} / f_{cd,_{fat}} + 0.43 \sqrt{1 - \sigma_{cd,_{min,_{equ}}} / \sigma_{cd,_{max,_{equ}}}} \right] \leq 1.0 \)
design value of compression strength \( f_{cd,_{fat}} = 15.00 \text{ N/mm}^2 \) at \( t_0 = 28 \text{ d} \)
material safety \( \gamma_c,_{fat} = 1.50 \)
load: \( N_{s1} = 0.00 \text{ kN} \quad M_{s1} = 500.00 \text{ kNm} \quad V_{s1} = 50.00 \text{ kN} \)
\( N_{s2} = 10.00 \text{ kN} \quad M_{s2} = 355.00 \text{ kNm} \quad V_{s2} = 45.00 \text{ kN} \)
reinforcement (initial state): \( A_{So} = 1.56 \text{ cm}^2 \quad A_{Su} = 15.61 \text{ cm}^2 \quad a_{s,_{bov}} = 2.05 \text{ cm}^2 / \text{m} \)

fatigue design for steel:

- concrete fatigue design:
  \( \sigma_{cd,_{min,_{equ}}} = 7.90 \text{ N/mm}^2 \)
- \( \Delta \sigma_{so,_{equ}} = -31.15 - 45.08 = 13.93 \text{ N/mm}^2 \)
- \( \Delta \sigma_{so,_{equ}} = 490.71 - 350.17 = 140.54 \text{ N/mm}^2 \)
- \( U_{ci} = 0.96 < 1.00 \Rightarrow \text{verification executed!} \)

- end state:
  \( \sigma_{cd,_{min,_{equ}}} = 0.97 \text{ N/mm}^2 \)
  \( \sigma_{cd,_{max,_{equ}}} = 1.08 \text{ N/mm}^2 \)
  \( U_{ci} = 0.10 < 0.54 \Rightarrow \text{verification executed!} \)

\( \Rightarrow \text{no additional fatigue reinforcement!} \)

limitation of steel tension and concrete compression stresses (EC 2, 7.2)
- permitted tensile stress of rein. \( \sigma_{O} = 0.80 \cdot f_{ck} = 400.0 \text{ N/mm}^2 \)
- permitted concrete compression stress \( \sigma_{c} = 0.60 \cdot f_{ck} = -15.0 \text{ N/mm}^2 \)
- stress forces and moments: \( N_{s} = 0.00 \text{ kN} \quad M_{s} = 500.00 \text{ kNm} \)
- reinforcement (initial state): \( A_{So} = 1.56 \text{ cm}^2 \quad A_{Su} = 15.61 \text{ cm}^2 \)

maximal reinforcement tensile stresses \( \min \text{max} \text{al concrete compression stress} \)
- initial state:
  \( \sigma_{so} = -45.1 \text{ N/mm}^2 \quad \sigma_{su} = 490.7 \text{ N/mm}^2 \quad \sigma_{s} = -11.0 \text{ N/mm}^2 \)
- end state:
  \( \sigma_{so} = -43.6 \text{ N/mm}^2 < 400.0 \text{ N/mm}^2 \quad \sigma_{s} = -10.1 \text{ N/mm}^2 > -15.0 \text{ N/mm}^2 \)
  \( \sigma_{su} = 399.0 \text{ N/mm}^2 < 400.0 \text{ N/mm}^2 \)
  \( \Rightarrow \Delta A_{su} = 3.7 \text{ cm}^2 \)

\( \Rightarrow \text{incl. stress reinforcement:} \quad \min A_{So} = 1.6 \text{ cm}^2 \quad \min A_{Su} = 19.3 \text{ cm}^2 \)

total reinc.:
- total \( A_{So} = 1.6 \text{ cm}^2 \quad A_{Su} = 19.3 \text{ cm}^2 \)
- total \( a_{s,_{bov}} = 2.05 \text{ cm}^2 / \text{m} \)

degree of utilization: \( U = 0.82 \)

selected:
- Longitudinal, top: \( 2 \ 0 \ 10 = 1.6 \text{ cm}^2 \geq 1.6 \text{ cm}^2 \)
- bottom: \( 4 \ \phi \ 20 + 2 \ \phi \ 20 = 18.8 \text{ cm}^2 < 19.3 \text{ cm}^2 \)
- stirrups: 2-shear: \( \phi / 8 \ 30 = 3.35 \text{ cm}^2 / \text{m} > 2.05 \text{ cm}^2 / \text{m} \)

anchorage lengths top \( (A_{sb,_{min}} = 0.00 \text{ cm}^2 \quad A_{s,_{ext}} = 1.57 \text{ cm}^2) \):
- basic size of anchorage length, \( b_{min} \text{ minimum value of anchorage length, } b_{ext} \text{ intermediate support with hooks:} \)
  \( l_{b} = 57.7 \text{ cm} \quad l_{b,_{min}} = 12.1 \text{ cm} \quad l_{b,_{net}} = 12.1 \text{ cm} \)
  \( l_{b,_{dir}} = 8.1 \text{ cm} \quad l_{b,_{ind}} = 12.1 \text{ cm} \quad l_{b,_{int}} = 6.0 \text{ cm} \)
- without: \( l_{b} = 57.7 \text{ cm} \quad l_{b,_{min}} = 17.3 \text{ cm} \quad l_{b,_{net}} = 17.3 \text{ cm} \)
  \( l_{b,_{dir}} = 11.5 \text{ cm} \quad l_{b,_{ind}} = 17.3 \text{ cm} \quad l_{b,_{int}} = 6.0 \text{ cm} \)

anchorage lengths bottom \( (A_{sb,_{min}} = 15.61 \text{ cm}^2 \quad A_{s,_{ext}} = 18.85 \text{ cm}^2) \):
**reinforcement drawing:**

scale 1 : 20

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1.45

75

10

25
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**cross-section data**

- Gross area of concrete: $A_c = 30.8$ dm$^2$
- Second moment of area: $I_{zz} = 166.2$ dm$^4$
- Moment of resistance: $W_{zz} = 33.1$ dm$^3$
- Distance of centre of gravity from upper edge: $z_i = 24.8$ cm
- Total area of longitudinal reinforcement: $\Sigma(\text{min } A_s) = 20.9$ cm$^2$ $\Rightarrow$ $\rho_s = 0.68\% < 8.00\%$

**material properties for design calculation**

<table>
<thead>
<tr>
<th>Concrete</th>
<th>$f_{ck}$ (MN/m$^2$)</th>
<th>$\alpha$</th>
<th>$\varepsilon_{c(2)}$</th>
<th>$\varepsilon_{c(2u)}$</th>
<th>$\varepsilon_c$</th>
<th>$E_c$ (MN/m$^2$)</th>
<th>$f_{cm}$ (MN/m$^2$)</th>
<th>Reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>C25/30</td>
<td>25.0</td>
<td>0.850</td>
<td>-2.00</td>
<td>-3.50</td>
<td>2.00</td>
<td>31475.8</td>
<td>2.565</td>
<td>$f_y$</td>
</tr>
</tbody>
</table>

- Design value of compression strength $f_{cd} = f_{ck} \cdot \alpha / \gamma_c$
- Strain at reaching the maximum strength $\varepsilon_{c(2)}$, ult. compr. strain $\varepsilon_{c(2u)}$
- Conc. comp. stress $\sigma_c = f_{cd} (1+ (\gamma_c/\alpha))$ for $0 \leq \varepsilon_c \leq \varepsilon_{c(2)}$ and $\sigma_c = f_{cd}$ for $\varepsilon_c < \varepsilon_{c(2)}$
- Modulus of elasticity $E_{cm}$, mean value of axial tensile strength $f_{cm}$

- Design yield strength $f_{yd} = f_{y} / \gamma_y$
- Design tensile strength $f_{ts} = f_{t} / \gamma_y$
- Ult. tensile strain $\varepsilon_{su}$, modulus of elasticity $E_s$