POS. 22: DOPPEL-T (REINFORCED CONCRETE 1-Achs.)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)
uniaxial bending with/without axial force (4H-BETON version: 11/2007-41)

I-section

- \( h = 100.0 \) cm, \( b = 40.0 \) cm
- \( h_0 = 20.0 \) cm, \( b_0 = 160.0 \) cm
- \( h_u = 30.0 \) cm, \( b_u = 60.0 \) cm

edge distances of longit. rein.

- \( d_0 = 3.8 \) cm, \( d_u = 6.6 \) cm

material

- C25/30
- B500 (A)
- \( \gamma_s = 1.15 \), \( \gamma_c = 1.50 \)
- exposure class X0

detailing of reinforcement

- limitation of compression zone height to \( \lim \eta = 0.617 \)

min./max. reinforcement

- \( A_{so} \) (9.2.1.1, 9.5.2), \( \max \rho_s = 8.00\% \)

initial reinforcement

- \( A_{so} = 0.00 \) cm², \( A_{su} = 0.00 \) cm²
- \( A_{so} = 0.00 \) cm²/m

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)
with \( f_{cd} = \frac{f_{cd}}{\gamma_c} = 14.2 \) MN/m² and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with \( f_{yy} = f_{yk} / \gamma_y = 434.8 \) MN/m²
and \( f_{td} = f_{tk} / \gamma_k = 455.6 \) MN/m²

verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)
with \( f_{c} = f_{cm} = 33.0 \) MN/m² and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with \( f_{yy} = f_{yk} / \gamma_y = 629.0 \) MN/m² and \( \omega_k = 25\% \)

design calculation values and minimum reinforcement areas (EC 2, 6.1)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( N_{Ed} )</th>
<th>( M_{Ed} )</th>
<th>( \varepsilon_{2u} )</th>
<th>( \varepsilon_{2u} )</th>
<th>( \varepsilon_{1u} )</th>
<th>( \varepsilon_{1u} )</th>
<th>( \varepsilon_{1u} )</th>
<th>( \varepsilon_{1u} )</th>
<th>( \varepsilon_{1u} )</th>
<th>( \xi )</th>
<th>( \zeta )</th>
<th>( d )</th>
<th>( A_{so} )</th>
<th>( A_{su} )</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100.00</td>
<td>1500.00</td>
<td>-2.78</td>
<td>-1.65</td>
<td>25.00</td>
<td>26.96</td>
<td>0.10</td>
<td>0.96</td>
<td>93.4</td>
<td>---</td>
<td>37.51</td>
<td>305.43</td>
<td>-0.99</td>
<td>0.07</td>
<td>25.00</td>
</tr>
</tbody>
</table>

\( \omega_k = 3.50\% \); concr. strain in state of failure (fibre 2), \( \omega_{sy} = 25\% \); reinforcement strain in state of failure (fibre 1)

- \( x - \xi \): height of concr. comp. zone, \( z - \zeta \): lever arm of internal forces, \( d - h - d \): effective depth

9) minimum reinforcement acc. to 9.2.1.1

\[ \Rightarrow \text{longitudinal reinforcement: } \min A_{so} = 0.0 \text{ cm}^2, \min A_{su} = 37.5 \text{ cm}^2 \]

shear and bond design calculation (EC 2, 6.2 + 6.3)

minimum reinforcement acc. to 9.2.2.5), material quality as flexural reinfor.
- \( z = 0.9 \) d (6.2.3.1)), \( c_{v,0} = 3.0 \) cm, \( D \): compression reinforc.
- angle of reinforcement \( \alpha = 90.0^\circ \), angle of compr. strut \( \theta_{ew} = 0^\circ \)
- tension reinforcement \( A_{s1, ew} = 8.0 \) cm²
the minimum value of \( V_{dct} \) is limited acc. to design code (\( V_{dct} \geq V_{dct} \)).
only web design; connection of compression/tension boom has to be designed separately.

design calculation of shear force (EC 2, 6.2)

<table>
<thead>
<tr>
<th>( V_{Ed} )</th>
<th>( \rho_1 )</th>
<th>( \gamma )</th>
<th>( V_{dct} )</th>
<th>( \theta )</th>
<th>( \cot \theta )</th>
<th>( V_{dct} )</th>
<th>( A_B )</th>
<th>( \rho_s, \rho_v )</th>
<th>( a_s, b_v )</th>
<th>( \text{note} )</th>
</tr>
</thead>
</table>
| 1 | 50.00 | 0.21 | 84.1 | 104.46 | 18.4 | 3.00 | 1071.76 | 1 | 126.1 | 3.28 | minimum reinforcement

\( \rho \): ratio of longit. rein. related to static height, \( \gamma \): decisive inner lever arm
\( V_{dct} \): design value of shear resistance without shear reinforcement, \( \alpha \): angle of compr. strut,
\( V_{dct} \): design value of maximal shear resistance, \( a_B \): shift rule
\( A_B \): range of utilization see NA-DE

shear at the interface between concrete cast at different times (EC 2, 6.2.5)

design value of the shear stress in the interface \( \tau_{Ed,i} = \beta \cdot V_{Ed} / (b_1 z) \) with \( \beta = 1.00 \),
width of contact surface \( b_1 = 40.00 \) cm (in web), angle of compr. strut \( \theta_1 = 45^\circ \),
normal stress perpendicular to interface \( \sigma_n = 0 \)
interaxe stress from dynamic load
interface surface condition: smooth \( (\Rightarrow c = 0.10, \mu = 0.6) \)
<table>
<thead>
<tr>
<th>( v_E \cdot J )</th>
<th>( V_{Ku} \cdot J )</th>
<th>( Z_J )</th>
<th>( V_{Ku} \cdot \max \cdot J )</th>
<th>( a_s \cdot b_o )</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>59.48</td>
<td>0.00</td>
<td>84.1</td>
<td>566.67</td>
<td>1.90</td>
</tr>
</tbody>
</table>

analogous to shear analysis: \( V_{Ku} \cdot J \) \( = \) \( c_{ef} \cdot d + \mu \cdot c_{su} \cdot v_{Ku,\max} \cdot J \) \( = 0.5 \cdot v_{E} \cdot J \) (reduction of strength)
design value of concrete tensile strength: \( f_{\text{ctd}} = 1.20 \text{ N/mm}^2 \)

\[ \Rightarrow \text{shear reinforcement: } \min a_s, b_o = 3.28 \text{ cm}^2 / \text{m} = \max (a_s, b_o, V, a_s, b_o) \]

**crack control** (EC 2, 7.3.7: 7.3.2 minimum reinforcement, 7.3.4 crack control)
cracking in bending restraint (intrinsically imposed) **minimum reinforcement**
coeff. - stress distribution \( k_c = 0.53 / 0.35 \)
coeff. - self-equiv. stresses \( k = 0.74 \)
concr. tens. str. (restr.) \( f_{\text{ct}, \text{eff}} = 2.56 \text{ N/mm}^2 \)
tension zones \( a_{cto} = 16.6 \text{ dm}^2 \) \( a_{ctu} = 25.7 \text{ dm}^2 \)
\( (a_{sto, \min} = 5.1 \text{ cm}^2 \) \( a_{stu, \min} = 11.5 \text{ cm}^2 \)

**crack control**
concr. tens. strength (load) \( f_{\text{ct}, \text{eff}} = f_{\text{ctm}} = 2.56 \text{ N/mm}^2 \)
effective slab width \( b_{\text{eff}} = 51.4 / 59.8 \text{ cm} \)
\( \sigma_{so} = 0.0 \text{ N/mm}^2 \) \( \sigma_{su} = 385.7 \text{ N/mm}^2 \)
\( \sigma_{so} = 0.0 \text{ N/mm}^2 \) \( \varepsilon_s: \varepsilon_c = 0.000 \text{%} \) \( \sigma_{su} = 5.71 \text{ N/mm}^2 \) \( \varepsilon_s: \varepsilon_c = 1.747 \text{%} \)
\( \sigma_{cto} = 171.6 \text{ mm} \) \( \sigma_{ctu} = 0.0 \text{ cm}^2 (d_{so} = 20 \text{ mm}) \)
\( a_{sto, \text{ste}} = 37.5 \text{ cm}^2 \) \( a_{stu, \text{ste}} = 21.3 \text{ mm} \) \( w_u = 0.28 \text{ mm} \)
**additional reinforcement**
\( \max a_{sto} = 5.1 \text{ cm}^2 \) \( \Rightarrow a_{sto} = 5.1 \text{ cm}^2 \)

\[ \Rightarrow \text{incl. anti-crack reinforcement: } \min a_s = 5.1 \text{ cm}^2 \min a_{su} = 37.5 \text{ cm}^2 \]

**fatigue design** (EC 2, 6.6.8 + 6.8.7(2))
for steel: \( U_{s1} = \Delta \sigma \leq U_{s2} = 70.0 \text{ N/mm}^2 \)
stress range \( \Delta \sigma = \sigma_{so} - \sigma_{su} \)
shear force: \( U_{s1v} = \Delta \sigma_{sv} \leq U_{s2v} = 70.0 \text{ N/mm}^2 \)
for conc.: \( U_{c1} = |\sigma_{cd, \max} | / f_{\text{cd}, \text{fat}} \leq 0.5 + 0.45 \) \( |\sigma_{cd, \min} | / f_{\text{cd}, \text{fat}} \leq 0.9 \)
design value of compression strength \( f_{\text{cd}, \text{fat}} = 15.00 \text{ N/mm}^2 \) at \( t_0 = 28 \text{ d} \)
material safety \( \gamma_c = 1.50 \)
reduction factor of shear force \( a_c = 0.75 \) \( (f_{\text{cd}, \text{fat}} = a_c f_{\text{cd}, \text{fat}}) \)
load: \( N_s = 50.00 \text{ kN} \) \( M_s = 900.00 \text{ kNm} \) \( V_s = 50.00 \text{ kN} \)
\( N_s = 100.00 \text{ kN} \) \( M_s = 1350.00 \text{ kNm} \) \( V_s = 75.00 \text{ kN} \)
**reinforcement (initial state):** \( A_{so} = 5.12 \text{ cm}^2 \) \( A_{su} = 37.51 \text{ cm}^2 \) \( a_s, b_o = 3.28 \text{ cm}^2 / \text{m} \)
**fatigue design for steel:**
initial state:
\( \Delta \sigma_{so} = 39.79 - \varepsilon_{so} = 20.53 \text{ N/mm}^2 \)
\( \Delta \sigma_{so} = 417.85 - 276.72 = 141.13 \text{ N/mm}^2 \)
end state:
\( \Delta \sigma_{so} = -32.05 - -48.36 = 16.31 \text{ N/mm}^2 \)
\( U_{s1} = 16.31 < U_{s2} = 70.00 \Rightarrow \Delta \sigma_{so, \text{fat}} = 0.0 \text{ cm}^2 \)
\( \Delta \sigma_{su} = 207.24 - 137.31 = 69.93 \text{ N/mm}^2 \)
\( U_{s1} = 69.93 < U_{s2} = 70.00 \Rightarrow \Delta \sigma_{su, \text{fat}} = 39.6 \text{ cm}^2 \)
**reinforcement (shear force):**
\( \Delta \sigma_{sv} = 156.90 - 104.60 = 52.30 \text{ N/mm}^2 \)
\( U_{s1v} = 52.30 < U_{s2v} = 70.00 \)

\[ \Rightarrow \text{incl. fatigue reinforcement: } \min a_s = 5.1 \text{ cm}^2 \min a_{su} = 77.1 \text{ cm}^2 \]

**limitation of steel tension and concrete compression stresses** (EC 2, 7.2)
permitted tensile stress of reinf. \( \sigma_s = 0.80 \cdot f_y = 400.0 \text{ N/mm}^2 \)
permitted concrete compression stress \( \sigma_c = 0.60 \cdot f_{ck} = -15.0 \text{ N/mm}^2 \)
stress forces and moments: \( N_o = 100.00 \text{ kN} \) \( M_o = 1500.00 \text{ kNm} \)
**reinforcement (initial state):** \( A_s = 5.12 \text{ cm}^2 \) \( A_{su} = 77.14 \text{ cm}^2 \)
**maximal reinforcement tensile stresses**
initial state:
\( \sigma_{so} = -54.1 \text{ N/mm}^2 \) \( \sigma_{su} = 229.8 \text{ N/mm}^2 \)
= end state
minimal concrete compression stress
initial state:
\[ \sigma_{\text{c}} = -9.6 \, \text{N/mm}^2 \]
end state

⇒ no additional stress reinforcement!

total reinforcement:
\[ A_{\text{s0}} = 5.1 \, \text{cm}^2 \quad A_{\text{su}} = 77.1 \, \text{cm}^2 \]

degree of utilization:
\[ U = 0.51 \]

selected:
longitudinal, top: 2 \( \varnothing \, 10 = 1.6 \, \text{cm}^2 < 5.1 \, \text{cm}^2 \)
bottom: 8 \( \varnothing \, 20 + 4 \varnothing \, 20 = 37.7 \, \text{cm}^2 < 77.1 \, \text{cm}^2 \)

stirrups, 2-shear: \( \varnothing \, 8 / 30 \, \text{cm} = 3.35 \, \text{cm}^2/m > 3.28 \, \text{cm}^2/m \)

anchorage lengths top \( (A_{\text{s0, min}} = 0.00 \, \text{cm}^2 \quad A_{\text{s0, ext}} = 1.57 \, \text{cm}^2) \):
- \( b_0 \): basic size of anchorage length
- \( l_{\text{b, min}} \): minimum value of anchorage length
- \( l_{\text{b, ext}} \): anchorage length
curt. of longitudinal reinforcement: anch. l. at \( b_0 \), direct end support, \( b_{\text{n, int}} \): indirect end support, \( b_{\text{n, ext}} \): intermediate support

with hooks:
\[ l_0 = 57.7 \, \text{cm}, \quad l_{\text{b, min}} = 12.1 \, \text{cm}, \quad l_{\text{b, net}} = 12.1 \, \text{cm} \]
\[ l_{\text{b, dir}} = 8.1 \, \text{cm}, \quad l_{\text{b, ext}} = 6.0 \, \text{cm} \]

without:
\[ l_0 = 57.7 \, \text{cm}, \quad l_{\text{b, min}} = 17.3 \, \text{cm}, \quad l_{\text{b, net}} = 17.3 \, \text{cm} \]
\[ l_{\text{b, dir}} = 11.5 \, \text{cm}, \quad l_{\text{b, ext}} = 6.0 \, \text{cm} \]

anchorage lengths bottom \( (A_{\text{s0, min}} = 37.5 \, \text{cm}^2 \quad A_{\text{s0, ext}} = 37.7 \, \text{cm}^2) \):
- \( b_0 \): basic size of anchorage length
- \( l_{\text{b, min}} \): minimum value of anchorage length
- \( l_{\text{b, ext}} \): anchorage length
curt. of longitudinal reinforcement: anch. l. at \( b_0 \), direct end support, \( b_{\text{n, int}} \): indirect end support, \( b_{\text{n, ext}} \): intermediate support

with hooks:
\[ l_0 = 80.7 \, \text{cm}, \quad l_{\text{b, min}} = 20.0 \, \text{cm}, \quad l_{\text{b, net}} = 56.2 \, \text{cm} \]
\[ l_{\text{b, dir}} = 37.5 \, \text{cm}, \quad l_{\text{b, ext}} = 12.0 \, \text{cm} \]

without:
\[ l_0 = 80.7 \, \text{cm}, \quad l_{\text{b, min}} = 24.2 \, \text{cm}, \quad l_{\text{b, net}} = 80.3 \, \text{cm} \]
\[ l_{\text{b, dir}} = 53.5 \, \text{cm}, \quad l_{\text{b, ext}} = 12.0 \, \text{cm} \]

reinforcement drawing:
scale 1:30

\[ c_{\text{vo}} = 2.5 \, \text{cm} \]
\[ c_{\text{uv}} = 3.5 \, \text{cm} \]

cross-section data
- gross area of concrete: \( A_{\text{c}} = 70.0 \, \text{dm}^2 \)
- second moment of area: \( I_{\text{c5}} = 723.0 \, \text{dm}^4 \)
- moment of resistance: \( W_{\text{c5}} = 119.1 \, \text{dm}^3 \)
- distance of centre of gravity from upper edge: \( z_{\text{s}} = 39.3 \, \text{cm} \)
- total area of longitudinal reinforcement: \( \Sigma_{\text{min}} A_{\text{s}} = 82.3 \, \text{cm}^2 \)

material properties for design calculation

<table>
<thead>
<tr>
<th>concrete</th>
<th>( f_{\text{k}, \text{c}} )</th>
<th>( \alpha )</th>
<th>( e_{\text{c2}} )</th>
<th>( e_{\text{c2u}} )</th>
<th>( n_{\text{c}} )</th>
<th>( E_{\text{c}} )</th>
<th>( f_{\text{cm}} )</th>
<th>reinforcement</th>
<th>( f_{\text{yk}} )</th>
<th>( f_{\text{tk}} )</th>
<th>( e_{\text{su}} )</th>
<th>( E_{\text{s}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>C25/30</td>
<td>25.0</td>
<td>0.850</td>
<td>-2.00</td>
<td>-3.50</td>
<td>2.00</td>
<td>31475.8</td>
<td>2.565</td>
<td>BSt 500 (A)</td>
<td>500.0</td>
<td>525.0</td>
<td>25.00</td>
<td>200000.0</td>
</tr>
<tr>
<td>design value of compression strength ( f_{\text{cd}} = f_{\text{k}, \text{c}} / \gamma_{\text{c}} )</td>
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<tr>
<td>design yield strength ( f_{\text{yd}} = f_{\text{k}} / \gamma_{\text{y}} )</td>
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\( \gamma_{\text{c}} \) and \( \gamma_{\text{y}} \) are coefficients for modulus of elasticity \( E_{\text{cm}} \) and mean value of axial tensile strength \( f_{\text{cm}} \)