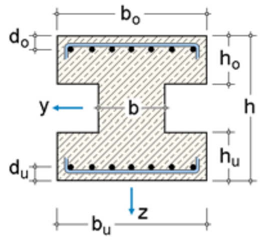
POS. 22: DOPPEL-T (REINFORCED CONCRETE 1-ACHS.)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)

uniaxial bending with/without axial force (4H-BETON version: 11/2007-4I)



I-section
h = 100.0 cm, b = 40.0 cm
h_o = 20.0 cm, b_o = 160.0 cm
h_u = 30.0 cm, b_u = 60.0 cm

edge distances of longit. reinf.
d_o = 3.8 cm, d_u = 6.6 cm

material
C25/30
BSt 500 (A)
γ_s = 1.15, γ_c = 1.50
exposure class X0

detailing of reinforcement
limitation of compression zone height
to lim ξ = 0.617

min./max. reinforcement
min A_s (9.2.1.1, 9.5.2), max ρ_0 = 8.00%

initial reinforcement
A_{s0o} = 0.00 cm^2, A_{s0u} = 0.00 cm^2
$a_{s0bü}$ = 0.00 cm^2/m

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)
with fcd = αc fck / γc = 14.2 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with fyd = fyk / γs = 434.8 MN/m2
and ftd = ftk / γs = 456.5 MN/m2 !
verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)
with fc = fcm = 33.0 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with fy = fyk, ft = 525.0 MN/m2 and εuk = 25‰ !

design calculation values and minimum reinforcement areas (EC 2, 6.1)

	γ	N_{Ed}	M_{Ed}	ε_{c2u}	ε_{s2u}	ε_{s1u}	ε_{c1u}	ξ	ζ	d	A_{so}	A_{su}	note
	-	kN	kNm	‰	‰	‰	‰	-	-	cm	cm^2	cm^2	
1	---	100.00	1500.00	-2.78	-1.65	25.00	26.96	0.10	0.96	93.4	----	**37.51**	
			305.43	-0.99	0.07	25.00	26.84	----	----	----	----	6.63	9)

ε_{c2u} = -3.50‰: concr. strain in state of failure (fibre 2), ε_{s1u} = 25.00‰: reinforcement strain in state of failure (fibre 1)
x = ξ d: height of conc. compr. zone, z = ζ d: lever arm of internal forces, d = h - d_1: effective depth

9) minimum reinforcement acc. to 9.2.1.1

⇒ longitudinal reinforcement: **min A_{so} = 0.0 cm^2** **min A_{su} = 37.5 cm^2**

shear and bond design calculation (EC 2, 6.2 + 6.3)
minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf.
z = 0.9 d (6.2.3(1)), $c_{v,D}$ = 3.0 cm, D = compression reinf.
angle of reinforcement α = 90.0°, angle of compr. strut θ_{gew} = 0°
tension reinforcement $A_{sl,gew}$ = **8.0** cm^2
the minimum value of V_{Rdct} is limited acc. to design code (V_{Rdct} ≥ min V_{Rdct}).
only web design; connection of compression/tension boom has to be designed separately.

design calculation of shear force (EC 2, 6.2)

	V_{Ed}	ρ_l	z	V_{Rdct}	θ	cot θ	V_{Rdmax}	AB	a_l	$a_{s,büv}$	note
	kN	%	cm	kN	°		kN		cm	cm^2/m	
1	50.00	0.21	84.1	104.46	18.4	3.00	1071.76	1	126.1	3.28	minimum reinforcement

ρ_l: ratio of longit. reinf. related to static height, z: decisive inner lever arm
V_{Rdct}: design value of shear resistance without shear reinforcement, θ: angle of compr. strut,
V_{Rdmax}: design value of maximal shear resistance, a_l: shift rule
AB: range of utilization see NA-DE

shear at the interface between concrete cast at different times (EC 2, 6.2.5)
design value of the shear stress in the interface $v_{Ed,j}$ = $\beta \cdot V_{Ed}$ / ($z \cdot b_j$) with β = 1.00,
width of contact surface b_j = 40.00 cm (in web), angle of compr. strut θ_j = 45°,
normal stress perpendicular to interface σ_n = 0
interface stress from dynamic load
interface surface condition: smooth (⇒ c = 0.10, μ = 0.6)

	$v_{Ed,j}$ kN/m²	$v_{Rdct,j}$ kN/m²	z_j cm	$v_{Rdmax,j}$ kN/m²	$a_{s,büj}$ cm²/m	note
1	59.48	0.00	84.1	566.67	1.90	

analogous to shear analysis: $v_{Rdct,j} = c \cdot f_{ctd} + \mu \cdot \sigma_n$, $v_{Rdmax,j} = 0.5\, \nu \cdot f_{cd}$ (ν reduction factor of strength)
design value of concrete tensile strength: $f_{ctd} = 1.20$ N/mm²

⇒ **shear reinforcement: min $a_{s,bü}$ = 3.28 cm²/m** = max($a_{s,büV}$, $a_{s,büj}$)

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.4 crack control)
cracking in bending restraint (intrinsically imposed)
factor for progress of hardening $k_{z,t}$ = 1.00
formation of first crack: N_{cr} = 100.00 kN
crack width w_k = 0.30 mm
sel. diameter d_{so} = 20 mm d_{su} = 20 mm
crack forces and moments:
N_r = 80.00 kN M_r = 1200.00 kNm
initial state: A_{so} = 0.00 cm² A_{su} = 37.51 cm²

minimum reinforcement:
coeff. - stress distribution k_c = 0.53 / 0.35
coeff. - self-equil. stresses k = 0.74
concr. tens. str. (restr.) $f_{ct,eff}$ = 2.56 N/mm²
tension zones A_{cto} = 16.6 dm² A_{ctu} = 25.7 dm²
($A_{sto,min}$ = 5.1 cm² $A_{stu,min}$ = 11.5 cm²)
crack control:
concr. tens. strength (load) $f_{ct,eff}$ = f_{ctm} = 2.56 N/mm²
effective slab width b_{eff} = 51.4 / 59.8 cm
σ_{so} = 0.0 N/mm² σ_{su} = 385.7 N/mm²
σ_{co} = 0.00 N/mm², $\varepsilon_s - \varepsilon_c$ = 0.000‰, $s_{r,max}$ = 0.0 mm
σ_{cu} = 5.71 N/mm², $\varepsilon_s - \varepsilon_c$ = 1.747‰, $s_{r,max}$ = 171.6 mm
($A_{sto,ste}$ = 0.0 cm² (d_{so} = 20 mm)
$A_{stu,ste}$ = 37.5 cm² (⇒ d_{su} = 21.3 mm > 20, w_u = 0.28 mm)
additional reinforcement:
max A_{sto} = 5.1 cm² ⇒ ΔA_{sto} = 5.1 cm²

⇒ **incl. anti-crack reinforcement: min A_{so} = 5.1 cm² min A_{su} = 37.5 cm²**

fatigue design (EC 2, 6.8.6 + 6.8.7(2))
for steel: $U_{s1} = \Delta\sigma_s \le U_{s2}$ = 70.0 N/mm²
stress range $\Delta\sigma_s = \sigma_{s,O} - \sigma_{s,U}$
shear force : $U_{s1v} = \Delta\sigma_{sv} \le U_{s2v}$ = 70.0 N/mm²
for conc.: $U_{c1} = |\sigma_{cd,max}|/f_{cd,fat} \le 0.5 + 0.45\, |\sigma_{cd,min}|/f_{cd,fat} \le 0.9$
design value of compression strength $f_{cd,fat}$ = 15.00 N/mm² at t_0 = 28 d
material safety $\gamma_{c,fat} = \gamma_c$ = 1.50
reduction factor of shear force α_c = 0.75 ($f_{cdv,fat} = \alpha_c\, f_{cd,fat}$)
load: N_{s1} = 50.00 kN M_{s1} = 900.00 kNm V_{s1} = 50.00 kN
N_{s2} = 100.00 kN M_{s2} = 1350.00 kNm V_{s2} = 75.00 kN
reinforcement (initial state): A_{so} = 5.12 cm² A_{su} = 37.51 cm² $a_{s,büV}$ = 3.28 cm²/m

fatigue design for steel:
initial state:
$\Delta\sigma_{s0o}$ = -39.79 - -60.32 = 20.53 N/mm²
$\Delta\sigma_{s0u}$ = 417.85 - 276.72 = 141.13 N/mm²
end state:
$\Delta\sigma_{so}$ = -32.05 - -48.36 = 16.31 N/mm²
U_{s1o} = 16.31 < U_{s2} = 70.00 ⇒ $\Delta A_{so,fat}$ = 0.0 cm²
$\Delta\sigma_{su}$ = 207.24 - 137.31 = 69.93 N/mm²
U_{s1u} = 69.93 < U_{s2} = 70.00 ⇒ $\Delta A_{su,fat}$ = 39.6 cm²
reinforcement (shear force):
$\Delta\sigma_{sv}$ = 156.90 - 104.60 = 52.30 N/mm²
U_{s1v} = 52.30 < U_{s2v} = 70.00

concrete fatigue design:
$\sigma_{cd,min}$ = 5.89 N/mm²
$\sigma_{cd,max}$ = 8.67 N/mm²
U_{c1} = 0.58 < 0.68 < 0.9 ⇒ verification executed !
verification of compression strut:
$\sigma_{cdv,min}$ = 0.50 N/mm²
$\sigma_{cdv,max}$ = 0.74 N/mm²
U_{c1v} = 0.07 < 0.52 < 0.9 ⇒ verification executed !

⇒ **incl. fatigue reinforcement: min A_{so} = 5.1 cm² min A_{su} = 77.1 cm²**

limitation of steel tension and concrete compression stresses (EC 2, 7.2)
permitted tensile stress of reinf. $\sigma_s = 0.80 \cdot f_{yk}$ = 400.0 N/mm²
permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck}$ = -15.0 N/mm²
stress forces and moments: N_σ = 100.00 kN, M_σ = 1500.00 kNm
reinforcement (initial state): A_{so} = 5.12 cm² A_{su} = 77.14 cm²
maximal reinforcement tensile stresses
initial state:
σ_{0so} = -54.1 N/mm² σ_{0su} = 229.8 N/mm²
= end state

minimal concrete compression stress
initial state:
σ_{0c} = -9.6 N/mm²
= end state

⇒ no additional stress reinforcement !

total reinforc.: total A_{so} = 5.1 cm² A_{su} = 77.1 cm²
total $a_{s,büV}$ = 3.28 cm²/m
degree of utilization: U = 0.51

selected:

longitudinal, top : 2 Ø 10 = 1.6 cm² < 5.1 cm²
bottom:8 Ø 20 + 4 Ø 20 = 37.7 cm² < 77.1 cm²
stirrups, 2-shear: Ø 8 / 30 cm = **3.35 cm²/m** > 3.28 cm²/m

anchorage lengths top ($A_{sb,min}$ = 0.00 cm² $A_{s,exis}$ = 1.57 cm²):
l_b: basic size of anchorage length, $l_{b,min}$: minimum value of anchorage length, $l_{b,net}$: anchorage length
curt. of longit. tension reinf.: anch. l. at $l_{b,dir}$: direct end support, $l_{b,ind}$: indirect end support, $l_{b,Zwi}$: intermediate support

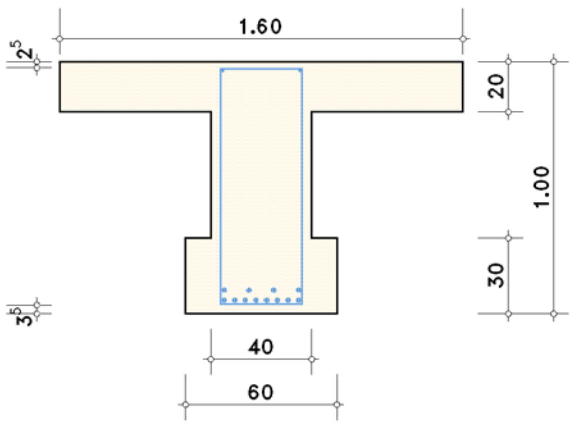
with hooks:	l_b = 57.7 cm,	$l_{b,min}$ = 12.1 cm,	$l_{b,net}$ = 12.1 cm
	$l_{b,dir}$ = 8.1 cm,	$l_{b,ind}$ = 12.1 cm,	$l_{b,Zwi}$ = 6.0 cm
without:	l_b = 57.7 cm,	$l_{b,min}$ = 17.3 cm,	$l_{b,net}$ = 17.3 cm
	$l_{b,dir}$ = 11.5 cm,	$l_{b,ind}$ = 17.3 cm,	$l_{b,Zwi}$ = 6.0 cm

anchorage lengths bottom ($A_{sb,min}$ = 37.51 cm² $A_{s,exis}$ = 37.70 cm²):
l_b: basic size of anchorage length, $l_{b,min}$: minimum value of anchorage length, $l_{b,net}$: anchorage length
curt. of longit. tension reinf.: anch. l. at $l_{b,dir}$: direct end support, $l_{b,ind}$: indirect end support, $l_{b,Zwi}$: intermediate support

with hooks:	l_b = 80.7 cm,	$l_{b,min}$ = 20.0 cm,	$l_{b,net}$ = 56.2 cm
	$l_{b,dir}$ = 37.5 cm,	$l_{b,ind}$ = 56.2 cm,	$l_{b,Zwi}$ = 12.0 cm
without:	l_b = 80.7 cm,	$l_{b,min}$ = 24.2 cm,	$l_{b,net}$ = 80.3 cm
	$l_{b,dir}$ = 53.5 cm,	$l_{b,ind}$ = 80.3 cm,	$l_{b,Zwi}$ = 12.0 cm

reinforcement drawing:
scale 1 : 30

cvo = 2.5 cm
cvu = 3.5 cm



cross-section data
gross area of concrete: A_c = 70.0 dm², second moment of area: I_{cs} = 723.0 dm⁴
moment of resistance: W_{cs} = 119.1 dm³, distance of centre of gravity from upper edge: z_s = 39.3 cm
total area of longitudinal reinforcement: Σ(min A_s) = 82.3 cm² ⇒ ρ_s = 1.18% < 8.00%
material properties for design calculation

concrete	f_{ck}	α	ε_{c2}	ε_{c2u}	n_c	E_{cm}	f_{ctm}
	MN/m²	-	‰	‰	-	MN/m²	MN/m²
C25/30	25.0	0.850	-2.00	-3.50	2.00	31475.8	2.565

design value of compression strength $f_{cd} = \alpha_c f_{ck} / \gamma_c$
strain at reaching the maximum strength ε_{c2}, ult. compr. strain ε_{c2u}
concr. comp. stress $\sigma_c = f_{cd} (1-(1-\varepsilon_c/\varepsilon_{c2})^n)$ for $0 \geq \varepsilon_c > \varepsilon_{c2}$ and $\sigma_c = f_{cd}$ for $\varepsilon_{c2} \geq \varepsilon_c > \varepsilon_{c2u}$
modulus of elasticity E_{cm}, mean value of axial tensil strength f_{ctm}

reinforcem.	f_{yk}	f_{tk}	ε_{su}	E_s
	MN/m²	MN/m²	‰	MN/m²
BSt 500 (A)	500.0	525.0	25.00	200000.0

design yield strength $f_{yd} = f_{yk} / \gamma_s$
design tensile strength $f_{td} = f_{tk} / \gamma_s$
ult. tensile strain ε_{su}, modulus of elasticity E_s