POS. 25: CIRCLE (REINFORCED CONCRETE)

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)
uniaxial bending with/without axial force (4H-BETON version: 11/2007-4)

circular cross section
diameter

$\phi = 40.0 \text{ cm}$
edge distances of longitudinal reinforcement

distance

$\rho = 5.5 \text{ cm}$
material

C25/30
Bst 500 (A)

$\gamma_s = 1.15$, $\gamma_c = 1.50$
exposure class X0

detailing of reinforcement

circumferential reinforcement

min./max. reinforcement

$A_{S\rho} = 0.00 \text{ cm}^2$
$A_{S\phi} = 0.00 \text{ cm}^2$

verify in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)

with $f_{cd} = \frac{f_{ck}}{\gamma_c} = 14.2 \text{ MN/m}^2$ and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $f_{yd} = \frac{f_{yk}}{\gamma_y} = 484.8 \text{ MN/m}^2$
and $f_t = \frac{f_{tk}}{\gamma_t} = 456.5 \text{ MN/m}^2$

verify in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)

with $f_c = f_{cm} = 33.0 \text{ MN/m}^2$ and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with $f_{yd} = f_{yk}$, $f_t = 525.0 \text{ MN/m}^2$ and $\gamma_y = 25\%$

design calculation values and minimum reinforcement areas (EC 2, 6.1)

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$N_{Ed}$</th>
<th>$M_{Ed}$</th>
<th>$\varepsilon_{2u}$</th>
<th>$\sigma_{2u}$</th>
<th>$\varepsilon_{5u}$</th>
<th>$\sigma_{5u}$</th>
<th>$\varepsilon_{cl}$</th>
<th>$\sigma_{cl}$</th>
<th>$\zeta$</th>
<th>$d$</th>
<th>$D_b$</th>
<th>$S_A$</th>
<th>$A_{S\rho}$</th>
<th>$A_{S\phi}$</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>---</td>
<td>-1500.0</td>
<td>126.00</td>
<td>-3.50</td>
<td>-2.82</td>
<td>0.77</td>
<td>1.45</td>
<td>0.85</td>
<td>0.60</td>
<td>33.2</td>
<td>37.8</td>
<td>---</td>
<td>24.82</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\varepsilon_{2u} = -3.50\%$; concr. strain in state of failure (fibre 2), $\varepsilon_{5u} = 25.00\%$; reinforcement strain in state of failure (fibre 1)

$\sigma_{2u} = -3.50\%$; concr. strain in state of failure (fibre 2), $\sigma_{5u} = 25.00\%$; reinforcement strain in state of failure (fibre 1)

$X = \zeta$; height of concr. compr. zone, $z = \zeta$; lever arm of internal forces, $d$; effective depth, $D_b$; effective cross section width

$b_N$; acc. to NABau (01/05): smallest cross section width perpendicular to inner lever arm $z$ in height of internal forces

$\Rightarrow$ longitudinal reinforcement: $min A_{S\rho} = 24.82 \text{ cm}^2$

shear design calculation (EC 2, 6.2 + 6.3)

minimum reinforcement acc. to 9.2.2(5), material quality as flexural reinf.

$z = 0.9 d (6.2.3(1))$, $c_v, D = 3.0 \text{ cm}$, $D =$ compression reinf.

angle of reinforcement $\alpha = 90.0^\circ$, angle of compr. strut $\theta_{2\text{dew}} = 0^\circ$, efficiency factor $\alpha_k = 0.90$

the minimum value of $V_{\text{dew}}$ is limited acc. to design code ($V_{\text{dew}} < \gamma V_{\text{dew}}$).

design calculation of shear force (EC 2, 6.2)

<table>
<thead>
<tr>
<th>$V_{Ed}$</th>
<th>$p_t$</th>
<th>$\theta$</th>
<th>$\theta_{2\text{dew}}$</th>
<th>$V_{\text{dew}}$</th>
<th>$V_{\text{lim}}$</th>
<th>$A_B$</th>
<th>$A_{S\beta}$</th>
<th>$A_{S\beta \text{dew}}$</th>
<th>note</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>250.0</td>
<td>1.00</td>
<td>27.2</td>
<td>124.51</td>
<td>27.8</td>
<td>1.90</td>
<td>451.13</td>
<td>2</td>
<td>25.8</td>
</tr>
</tbody>
</table>

$\theta = \text{ratio of longit. reinf. related to static height, } \theta = \text{ effective inner lever arm}$

$V_{\text{dew}}$; design value of shear resistance without shear reinforcement, $\theta$: angle of compr. strut, $V_{\text{lim}}$: design value of maximal shear resistance, $\alpha_k$: shift rule

A; range of utilization see NA-DE

$\Rightarrow$ shear reinforcement: $min a_{S\beta, \text{dew}} = 12.36 \text{ cm}^2$/m

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)

cracking due to centr. restraint (intrinsically imposed) minimum reinforcement:

factor for progress of hardening $k_{2,t} = 1.00$

$\nu = 0.25 \text{ mm}$
sel. diameter $d_{\beta} = 10 \text{ mm}$

$N_F = -1700.00 \text{ kN}$, $N_C = 220.00 \text{ kNm}$

initial state: $A_{S\beta} = 24.82 \text{ cm}^2$

coeff. - stress distribution $k_c = 1.00$

coeff. - self-equil. stresses $k = 0.77$

$\text{conc. tens. str. (restr.) } f_{ct, \text{eff}} = 2.56 \text{ N/mm}^2$

tension zone $A_{CT\beta} = 6.3 \text{ dm}^2$

$\text{conc. tens. strength (load) } f_{ct, \text{eff}} = f_{ctm} = 2.56 \text{ N/mm}^2$

$\Rightarrow$ no additional anti-crack reinforcement!

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fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: \( U_{s1} = \gamma_f \cdot \sigma_{eq,\,fat} \geq U_{s2} = \Delta \sigma_{sk} (N^*) / \gamma_y, \, \sigma_{fat} = 152.17 \ \text{N/mm}^2 \)

damage equivalent stress range \( \Delta \sigma_{eq} = \sigma_{eq,0} - \sigma_{eq,1} \)

partial safety factors \( \gamma_f, \sigma_{eq,\,fat} = 1.00, \ \gamma_y, \sigma_{eq,\,fat} = 1.00, \ \gamma_y, \sigma_{fat} = 1.15 \)

allowable stress range \( \Delta \sigma_{sk} (N^*) = 175.0 \ \text{N/mm}^2 \)

shear force : \( \Delta \sigma_{skv} (N^*) = 107.0 \ \text{N/mm}^2 \implies U_{s2v} = \Delta \sigma_{skv} (N^*) / \gamma_y, \sigma_{fat} = 93.04 \ \text{N/mm}^2 \)

for conc.: \( U_{c1} = |\sigma_{cd,\,min,\,equ} | / f_{cd,\,fat} + 0.43 \cdot \sqrt{\text{tr}(1 - \sigma_{cd,\,min,\,equ} / \sigma_{cd,\,max,\,equ} )} \leq 1.0 \)

design value of compression strength \( f_{cd,\,fat} = 15.00 \ \text{N/mm}^2 \) at \( t_0 = 28 \ \text{d} \)

material safety \( \gamma_y, \sigma_{fat} = \gamma_y = 1.50 \)

load: \( N_{s1} = -1200.00 \ \text{kN} \quad M_{s1} = 86.00 \ \text{kNm} \quad V_{s1} = 250.00 \ \text{kN} \)

\( N_{s2} = -1450.00 \ \text{kN} \quad M_{s2} = 105.00 \ \text{kNm} \quad V_{s2} = 145.00 \ \text{kN} \)

reinforcement (initial state): \( A_{sb} = 24.82 \ \text{cm}^2 \quad a_{sb,\,bvy} = 12.36 \ \text{cm}^2 / \text{m} \)

fatigue design for steel: \( A_{sb} = 24.82 \ \text{cm}^2 \quad a_{sb,\,bvy} = 12.36 \ \text{cm}^2 / \text{m} \)

concrete fatigue design:

initial state:

\( \Delta \sigma_{eq,v} = -126.13 \ -158.90 = 32.77 \ \text{N/mm}^2 \)

end state:

\( \Delta \sigma_{eq,v} = 203.15 \ - 110.10 = 93.04 \ \text{N/mm}^2 \quad U_{s2v} = 35.00 \ \text{cm}^2 / \text{m} \)

\( \Rightarrow \Delta \sigma_{sb,fat} = 35.00 \ \text{cm}^2 / \text{m} \)

\( \Rightarrow \text{incl. fatigue reinforcement: } \min A_{sb} = 24.8 \ \text{cm}^2 \quad \min a_{sb,\,bvy} = 47.37 \ \text{cm}^2 / \text{m} \)

\( \Rightarrow \text{fatigue design for concrete not complied!} \)

Limitation of steel tension and concrete compression stresses (EC 2, 7.2)

permitted tensile stress of reinf. \( \sigma_s = 0.8 \cdot f_y = 400.0 \ \text{N/mm}^2 \)

permitted concrete compression stress \( \sigma_c = 0.6 \cdot f_{ck} = -15.0 \ \text{N/mm}^2 \)

stress forces and moments: \( N_0 = -1500.00 \ \text{kN} \quad M_0 = 126.00 \ \text{kNm} \)

reinforcement (initial state): \( A_{sb} = 24.82 \ \text{cm}^2 \)

maximal reinforcement tensile stresses \( \min \sigma_{sb} \)

initial state:

\( \sigma_{sb} = 44.1 \ \text{N/mm}^2 \quad \sigma_{sb} = -26.5 \ \text{N/mm}^2 \)

end state:

\( \sigma_{sb} = 11.2 \ \text{N/mm}^2 < 400.0 \ \text{N/mm}^2 \quad \sigma_{sb} = -14.8 \ \text{N/mm}^2 > -15.0 \ \text{N/mm}^2 \)

\( \Rightarrow \Delta \sigma_{sb} = 170.8 \ \text{cm}^2 \)

\( \Rightarrow \text{incl. stress reinforcement: } \min A_{sb} = 195.6 \ \text{cm}^2 \) (max \( \rho_0 \) !)

Fire protection acc. to EC 2, Teil 1-2 (10.06)

Mod. zone method (10 zones)

column flame application from all sides, fire duration 90 min

convective coeff. of thermal transfer \( \alpha = 25.0 \ \text{W/m}^2 \cdot \text{K} \), emissivity coeff. for concrete surface \( \varepsilon = 0.70 \)

normal dens. concr. with silicious aggr., moisture content 1.5%, upper limit of thermal conduct.

hot rolled reinforcing steel, density (reinforced concrete) \( p_c = 2300 \ \text{kg/m}^3 \)

assumption for the design calculation: concrete temperature of the coldest cross-section point (point M)

assumption for the design calculation: no inner stresses to be taken into account

assumption for the design calculation: stress-strain relation form acc. to EC 2 (fire case)

Simplified method for transient heat transport

finite volume method with explicit time integration taking into account non-linear material and boundary conditions
temperature profile (90 min, rotated 0°):
\[ dx = 0.50 \text{ cm} \quad dy = 0.50 \text{ cm} \quad (6661 \text{ cell nodes}), \quad \text{min} \; \Delta t = 0.055 \text{ min} \]

horizontal section through point M:

\[ \text{temperature in °C} \]
\[ \text{max: 975.24°C} \]
\[ \text{min: 86.72°C} \]

temperatures for 10 zones with related reduction factors:
\[ \Theta_1 = 826.1^\circ \text{C}, \; k_{c1} = 0.132 \quad \Theta_2 = 588.0^\circ \text{C}, \; k_{c2} = 0.468 \quad \Theta_3 = 425.0^\circ \text{C}, \; k_{c3} = 0.712 \]
\[ \Theta_4 = 311.2^\circ \text{C}, \; k_{c4} = 0.839 \quad \Theta_5 = 229.3^\circ \text{C}, \; k_{c5} = 0.921 \quad \Theta_6 = 169.8^\circ \text{C}, \; k_{c6} = 0.965 \]
\[ \Theta_7 = 129.2^\circ \text{C}, \; k_{c7} = 0.985 \quad \Theta_8 = 105.6^\circ \text{C}, \; k_{c8} = 0.997 \quad \Theta_9 = 93.6^\circ \text{C}, \; k_{c9} = 1.000 \]
\[ \Theta_{10} = 87.8^\circ \text{C}, \; k_{c10} = 1.000 \]

mean reduction factor (related temperature): \( k_{cm} = 0.802 \) (\( \Theta_{cm} = 348.1^\circ \text{C} \))

mean temperature in point M with related reduction factors: \( \Theta_{cm} = 86.7^\circ \text{C}, \; k_{cm} = 1.000 \)

static ineffective concrete boundary zone: \( \phi_z = 5.10 \text{ cm} \)

concrete temperature (design calculation) with associated reduction factor: \( \Theta_c = 86.7^\circ \text{C}, \; k_c = 1.000 \)

reinforcement temperatures: \( \Theta_{ba} = 395.5^\circ \text{C} \)

associated reduction factors: \( k_{s_y,a} = 1.000 \quad k_{s_p,a} = 0.429 \quad k_{s_s,a} = 0.705 \)

fire protection for \( \gamma_c = \gamma_s = 1 \) (parameters of stress-strain relation acc. to 3.2)

reduced cross-section radius: \( r = 14.90 \text{ cm} \)

design calculation values: \( N_{Ed,r1} = -1500.00 \text{ kN} \quad M_{Ed,r1} = 126.00 \text{ kNm} \)

material properties:

concr. \( \bar{\varepsilon_c} = 87.0^\circ \text{C} \):
\[ f_{c,\varepsilon} = 25.0 \text{ N/mm}^2 \quad (E_{c,\varepsilon} = 31475.8 \text{ N/mm}^2) \]
\[ e_{c,\varepsilon} = 3.75\% \quad e_{c,\varepsilon} = 0.00\% \]

reinf. \( \bar{\varepsilon_s} = 395^\circ \text{C} \):
\[ f_{s,\varepsilon} = 214.3 \text{ N/mm}^2 \quad f_{s,\varepsilon} = 500.0 \text{ N/mm}^2 \quad E_{s,\varepsilon} = 140906.1 \text{ N/mm}^2 \]
\[ e_{s,\varepsilon} = 5.2\% \quad e_{s,\varepsilon} = 20.0\% \quad e_{s,\varepsilon} = 50.0\% \quad e_{s,\varepsilon} = 0.00\% \]

\[ \Rightarrow \text{fire reinforcement:} \quad \text{min} \; A_{st} = 46.12 \text{ cm}^2 \]

total reinforcement: \( A_{st} = 195.6 \text{ cm}^2 \) (max \( p_0 \) !)

total \( A_{st,ov} = 47.37 \text{ cm}^2 / \text{m} \)

fatigue design for concrete not compiled!

selected: longitudinal, outer face: \( 8 \; \phi = 25.1 \text{ cm}^2 < 195.6 \text{ cm}^2 \)

stirrups, outer face: \( \phi = 10 / 30 \text{ cm} = 5.24 \text{ cm}^2 / \text{m} < 47.37 \text{ cm}^2 / \text{m} \)

anchorage lengths outer face (\( A_{st,\min} = 24.82 \text{ cm}^2 \quad A_{s,\min} = 25.13 \text{ cm}^2 \)):

- basic size of anchorage length, \( b_{\min} \): minimum value of anchorage length, \( b_{\min} \): anchorage length
- curb. of longit. tension ref.: anch. 1 at \( b_{s1} \): direct end support, \( b_{s2} \): indirect end support, \( b_{s3} \): intermediate support

with hooks:
\[ b = 80.7 \text{ cm}, \quad b_{s1} = 20.0 \text{ cm}, \quad b_{s2} = 55.8 \text{ cm} \]
\[ b_{s3} = 37.2 \text{ cm}, \quad b_{s4} = 55.8 \text{ cm}, \quad b_{s5} = 12.0 \text{ cm} \]

without:
\[ b = 80.7 \text{ cm}, \quad b_{s1} = 24.2 \text{ cm}, \quad b_{s2} = 79.7 \text{ cm} \]
\[ b_{s3} = 53.1 \text{ cm}, \quad b_{s4} = 79.7 \text{ cm}, \quad b_{s5} = 12.0 \text{ cm} \]
cross-section data
- Gross area of concrete: $A_c = 12.6 \, \text{dm}^2$
- Second moment of area: $I_{cs} = 12.6 \, \text{dm}^4$
- Distance of centre of gravity from upper edge: $z_s = 20.0 \, \text{cm}$
- Total area of longitudinal reinforcement: $\sum (\min A_s) = 195.6 \, \text{cm}^2 \Rightarrow \rho_s = 15.56\% > 8.00\%$

Material properties for design calculation

<table>
<thead>
<tr>
<th>Material</th>
<th>$f_{ck}$</th>
<th>$\alpha$</th>
<th>$\varepsilon_{c2}$</th>
<th>$\varepsilon_{c2u}$</th>
<th>$n_c$</th>
<th>$E_{cm}$</th>
<th>$f_{ctm}$</th>
<th>Reinforcement</th>
<th>$f_{yk}$</th>
<th>$f_{tk}$</th>
<th>$\varepsilon_{su}$</th>
<th>$E_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C25/30</td>
<td>25.0</td>
<td>0.850</td>
<td>-2.00</td>
<td>-3.50</td>
<td>2.00</td>
<td>31475.8</td>
<td>2.565</td>
<td>BSt 500 (A)</td>
<td>500.0</td>
<td>525.0</td>
<td>25.0</td>
<td>20000.0</td>
</tr>
</tbody>
</table>

Symbols: positive result values marked with -1.0 or **** in tables refer to incorrect resp. not computable conditions!