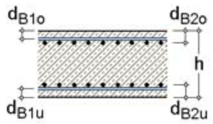
POS. 2: SLAB DESIGN

bending and shear design calculation (EC 2 (1.11), NA: Deutschland)

slab design (bending without axial force) (4H-BETON version: 11/2007-4q)



slab thickness
h = 30.0 cm
edge distances of longit. reinf.
d_{B1o} = 3.0 cm, d_{B2o} = 3.8 cm
d_{B1u} = 3.0 cm, d_{B2u} = 3.8 cm
transformation of design values
acc. to T.Baumann, 1972
material
C20/25
bottom:BSt 500 (A)
top : BSt 500 (A)
γ_s = 1.15, γ_c = 1.50

detailing of reinforcement orthogonal
B1 and B2 axially parallel
min./max./transverse reinforcement
min a_s acc. to 9.6.2, max ρ_0 = 8.00%
no secondary reinforcement direction
initial reinforcement
a_{s0B1o} = 4.24 cm²/m, a_{s0B2o} = 4.24 cm²/m
a_{s0B1u} = 2.57 cm²/m, a_{s0B2u} = 2.57 cm²/m
$a_{s0bü}$ = 0.00 cm²/m²

verifications in ultimate limit states are executed with stress-strain relation for concrete acc. to 3.1.7 (figure 3.3)
with fcd = αc fck / γc = 11.3 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (fig. 3.8) with fyd = fyk / γs = 434.8 MN/m2
and ftd = ftk / γs = 456.5 MN/m2 !
verifications in serviceability limit states are executed with stress-strain relation for concrete acc. to 3.1.5 (figure 3.2)
with fc = fcm = 28.0 MN/m2 and reinforcement stress-strain relation acc. to 3.2.7 (figure 3.8) with fy = fyk, ft = 525.0 MN/m2 and εuk = 25‰ !

design calculation values and minimum reinforcement areas (EC 2, 6.1)

output of the design relevant results for each reinforcement direction B1 top(o)/bottom(u) resp. B2 top(o)/bottom(u).

bwr	γ	m_{bwr}	ε_{c2u}	ε_{s2u}	ε_{s1u}	ε_{c1u}	z	a_{sB1o}	a_{sB1u}	a_{sB2o}	a_{sB2u}	note
	-	kNm/m	‰	‰	‰	‰	cm	cm²/m	cm²/m	cm²/m	cm²/m	
1 : $m_{xx,Ed}$ = -100.10 kNm/m, $m_{yy,Ed}$ = -34.00 kNm/m, $m_{xy,Ed}$ = -12.00 kNm/m												
B1o	---	-112.10	-3.50	-1.35	15.81	17.95	25.0	10.03	----			
B2o	---	-46.00	-2.34	1.63	25.00	28.97	25.3			3.98	----	
2 : $m_{xx,Ed}$ = -56.40 kNm/m, $m_{yy,Ed}$ = -11.30 kNm/m, $m_{xy,Ed}$ = -7.10 kNm/m												
B1o	---	-63.50	-2.91	0.20	25.00	28.10	25.9	5.38	----			
B2o	---	-18.40	-1.27	2.54	25.00	28.81	25.8			1.57	----	
minimum values:												
B1o		-33.16	-1.77	1.21	25.00	27.97	----	2.52	----			9)
B2o		-33.16	-1.84	2.05	25.00	28.89	----			2.60	----	9)

m_{bwr}: design moment transformed in the respective reinforcement direction (m_{xx}, m_{yy}, m_{xy} in kNm/m)
ε_{c2u} = -3.50‰: concr. strain in state of failure (fibre 2), ε_{s1u} = 25.00‰: reinforcement strain in state of failure (fibre 1)
z: lever arm of internal forces

9) minimum reinforcement acc. to 9.3.1.1

⇒ **longitudinal reinforcement: min a_{sB1o} = 10.03 cm²/m   min a_{sB2o} = 4.24 cm²/m**
(incl. initial reinf.)   **min a_{sB1u} = 2.57 cm²/m   min a_{sB2u} = 2.57 cm²/m**

shear and bond design calculation (EC 2, 6.2 + 6.3)

design calc. as 'slab', material quality as flexural reinf., z from bending design
angle of reinforcement α = 90.0°, angle of compr. strut θ_{gew} = 0°
design carried out for each reinforcement direction with subsequently summation of portions.
the minimum value of V_{Rdct} is limited acc. to design code (V_{Rdct} ≥ min V_{Rdct}).

design calculation of shear force (EC 2, 6.2)

	$v_{x,Ed}$	$v_{y,Ed}$	v_{Ed}	ρ_l	z	v_{Rdct}	θ	cot θ	v_{Rdmax}	AB	a_l	$a_{s,büv}$	note
	kN/m	kN/m	kN/m	%	cm	kN/m	°		kN/m		cm	cm²/m²	
1	128.20	21.30	128.20	0.37	25.0	107.26	18.4	3.00	636.58	1	37.4	4.24	min. reinf.
			21.30	0.16	25.3	105.18	18.4	3.00	646.05	1	38.0	0.00	min V_{Rdct}
2	74.30	11.40	74.30	0.20	25.9	107.26	18.4	3.00	659.66	1	38.8	0.00	min V_{Rdct}
			11.40	0.16	25.8	105.18	18.4	3.00	656.65	1	38.6	0.00	min V_{Rdct}

v_{Ed}: design shear force (je reinforcement direction), ρ_l reinf. ratio of longit. tension reinf. refering to static height, z = inner lever arm
v_{Rdct}: design value of shear resistance without shear reinforcement, θ: angle of compr. strut
v_{Rdmax}: design value of maximal shear resistance, a_l: shift rule
AB: range of utilization see NA-DE

shear at the interface between concrete cast at different times (EC 2, 6.2.5)

design value of the shear stress in the interface $v_{Ed,j} = \beta \cdot V_{Ed} / (z_j \cdot b_j)$ with $\beta = 1.00$

width of contact surface b_j = 100.00 cm, angle of compr. strut $\theta_j = 45°$

normal stress perpendicular to interface $\sigma_n = 0$

interface surface condition: smooth ⇒ c = 0.20, $\mu = 0.6$

	$v_{Ed,j}$ kN/m²	$v_{Rdct,j}$ kN/m²	z_j cm	$v_{Rdmax,j}$ kN/m²	$a_{s,büj}$ cm²/m²	note
1	513.5	206.3	25.0	1133.3	9.81	
	84.1	206.3	25.3	0.0	0.00	
2	287.2	206.3	25.9	1133.3	2.58	bond only
	44.3	206.3	25.8	0.0	0.00	

analogous to shear analysis: $v_{Rdct,j} = (c \cdot f_{ctd} + \mu \cdot \sigma_n) \cdot b_j$, $v_{Rdmax,j} = 0.5 \cdot \nu \cdot f_{cd} \cdot b_j$ (ν reduction factor of strength)

design value of concrete tensile strength: f_{ctd} = 1.03 N/mm²

⇒ **shear reinforcement: min $a_{s,bü}$ = 9.81 cm²/m²** = max($a_{s,büv}$, $a_{s,büj}$)

design action effects

verification notations: R = crack limitation S = fatigue design σ = stress verification

bwr	$m_{xx,Ed}$ m_{bwr} kNm/m	$m_{yy,Ed}$ kNm/m	$m_{xy,Ed}$ kNm/m
R	-68.60	-19.70	-7.40
B1o	-76.00		
B2o	-27.10		
S	-80.00	-24.00	-8.60

bwr	$m_{xx,Ed}$ m_{bwr} kNm/m	$m_{yy,Ed}$ kNm/m	$m_{xy,Ed}$ kNm/m
B1o	-88.60		
B2o	-32.60		
S	-59.80	-11.90	-6.50
B1o	-66.30		

bwr	$m_{xx,Ed}$ m_{bwr} kNm/m	$m_{yy,Ed}$ kNm/m	$m_{xy,Ed}$ kNm/m
B2o	-18.40		
σ	-80.00	-24.00	-8.60
B1o	-88.60		
B2o	-32.60		

crack control (EC 2, 7.3: 7.3.2 minimum reinforcement, 7.3.3 without direct calculation)

cracking in bending restraint (intrinsically imposed)

factor for progress of hardening $k_{z,t}$ = 1.00

formation of first crack: n_{cr} = 0.00 kN/m

crack width w_k = 0.30 mm

sel. diameter d_{sB1o} = 8 mm d_{sB2o} = 8 mm

d_{sB1u} = 8 mm d_{sB2u} = 8 mm

design action effects see above

reinforcement: a_{sB1o} = 10.03 a_{sB2o} = 4.24

in cm²/m a_{sB1u} = 2.57 a_{sB2u} = 2.57

minimum reinforcement:

coeff. - stress distribution k_c = 0.40

coeff. - self-equil. stresses k = 0.80

concr. tens. str. (restr.) $f_{ct,eff}$ = 2.21 N/mm²

($a_{stB1o,min}$ = 3.36 cm²/m $a_{stB2o,min}$ = 3.36 cm²/m

$a_{stB1u,min}$ = 3.36 cm²/m $a_{stB2u,min}$ = 3.36 cm²/m)

crack control:

concr. tens. strength (load) $f_{ct,eff} = f_{ctm}$ = 2.21 N/mm²

σ_{sB1o} = 301.6 N/mm² σ_{sB2o} = 255.6 N/mm²

σ_{sB1u} = 0.0 N/mm² σ_{sB2u} = 2.5 N/mm²

($a_{stB1o,ste}$ = 10.03 cm²/m (⇒ d_{so} = 9.6 mm > 8)

$a_{stB1u,ste}$ = 2.57 cm²/m (d_{su} = 8 mm)

$a_{stB2o,ste}$ = 4.24 cm²/m (⇒ d_{so} = 12.2 mm > 8)

$a_{stB2u,ste}$ = 2.57 cm²/m (⇒ d_{su} = 60.0 mm > 8))

additional reinforcement:

Δa_{stB1o} = 0.00 cm²/m Δa_{stB2o} = 0.00 cm²/m

Δa_{stB1u} = 0.79 cm²/m Δa_{stB2u} = 0.79 cm²/m

⇒ **incl. anti-crack reinforcement: min a_{sB1o} = 10.03 cm²/m min a_{sB2o} = 4.24 cm²/m**

min a_{sB1u} = 3.36 cm²/m min a_{sB2u} = 3.36 cm²/m

fatigue design (EC 2, 6.8.5 + 6.8.7(1))

for steel: $U_{s1} = \gamma_{F,fat}\ \gamma_{Ed,fat}\ \Delta\sigma_{s,equ} \le U_{s2} = \Delta\sigma_{Rsk}(N^*)/\gamma_{s,fat}$ = 73.91 N/mm²

damage equivalent stress range $\Delta\sigma_{s,equ} = \sigma_{s,O} - \sigma_{s,U}$

partial safety factors $\gamma_{F,fat}$ = 1.00, $\gamma_{Ed,fat}$ = 1.00, $\gamma_{s,fat} = \gamma_s$ = 1.15

allowable stress range $\Delta\sigma_{Rsk}(N^*)$ = 85.0 N/mm²

for conc.: $U_{c1} = |\sigma_{cd,max,equ}|/f_{cd,fat} + 0.43\ \mathrm{sqrt}(1 - \sigma_{cd,min,equ}/\sigma_{cd,max,equ}) \le 1.0$

design value of compression strength $f_{cd,fat}$ = 10.43 N/mm² at t_0 = 28 d

material safety $\gamma_{c,fat} = \gamma_c$ = 1.50

design action effects see above

reinforcement: a_{sB1o} = 10.03 cm²/m a_{sB2o} = 4.24 cm²/m a_{sB1u} = 3.36 cm²/m a_{sB2u} = 3.36 cm²/m

fatigue design for steel:

initial state:

$\Delta\sigma_{s0B1o,equ}$ = 351.90 - 262.98 = 88.92 N/mm²

$\Delta\sigma_{s0B1u,equ}$ = -28.56 - -39.33 = 10.77 N/mm²

$\Delta\sigma_{s0B2o,equ}$ = 307.54 - 173.41 = 134.13 N/mm²

$\Delta\sigma_{s0B2u,equ}$ = 2.71 - 1.82 = 0.89 N/mm²

end state:

$\Delta\sigma_{sB1o,equ}$ = 291.64 - 217.95 = 73.69 N/mm²

U_{s1B1o} = 73.69 < U_{s2} = 73.91 ⇒ $\Delta a_{sB1o,fat}$ = 2.14 cm²/m

$\Delta\sigma_{sB1u,equ}$ = -28.95 - -39.68 = 10.73 N/mm^2
U_{s1B1u} = 10.73 < U_{s2} = 73.91
$\Delta\sigma_{sB2o,equ}$ = 168.33 - 94.93 = 73.40 N/mm^2
U_{s1B2o} = 73.40 < U_{s2} = 73.91 ⇒ $\Delta a_{sB2o,fat}$ = 3.64 cm^2/m
$\Delta\sigma_{sB2u,equ}$ = -3.79 - -7.02 = 3.23 N/mm^2
U_{s1B2u} = 3.23 < U_{s2} = 73.91

concrete fatigue design:
$\sigma_{cdB1,min,equ}$ = 8.55 N/mm^2
$\sigma_{cdB1,max,equ}$ = 11.19 N/mm^2
U_{c1B1} = 1.28 > 1.00
$\sigma_{cdB2,min,equ}$ = 3.13 N/mm^2
$\sigma_{cdB2,max,equ}$ = 5.46 N/mm^2
U_{c1B2} = 0.80 < 1.00
⇒ verification **not** complied !

⇒ incl. fatigue reinforcement: min a_{sB1o} = 12.17 cm^2/m min a_{sB2o} = 7.88 cm^2/m
min a_{sB1u} = 3.36 cm^2/m min a_{sB2u} = 3.36 cm^2/m
⇒ fatigue design for concrete not complied !

limitation of steel tension and concrete compression stresses (EC 2, 7.2)
permitted tensile stress of reinf. $\sigma_s = 0.80 \cdot f_{yk}$ = 400.0 N/mm^2
permitted concrete compression stress $\sigma_c = 0.60 \cdot f_{ck}$ = -12.0 N/mm^2
design action effects see above
reinforcement: a_{sB1o} = 12.17 cm^2/m a_{sB2o} = 7.88 cm^2/m a_{sB1u} = 3.36 cm^2/m a_{sB2u} = 3.36 cm^2/m

maximal reinforcement tensile stresses
initial state:
σ_{0sB1o} = 291.6 N/mm^2 σ_{0sB2o} = 168.3 N/mm^2
σ_{0sB1u} = 0.0 N/mm^2 σ_{0sB2u} = 0.0 N/mm^2
= end state

minimal concrete compression stress
initial state:
σ_{0cB1} = -11.2 N/mm^2
σ_{0cB2} = -5.5 N/mm^2
= end state

⇒ no additional stress reinforcement !

total reinforc.: total a_{sB1o} = 12.17 cm^2/m a_{sB2o} = 7.88 cm^2/m
total a_{sB1u} = 3.36 cm^2/m a_{sB2u} = 3.36 cm^2/m
total $a_{s,büv}$ = 9.81 cm^2/m^2
degree of utilization: U = 0.83
height of compression zone: x_{min} = 71.2 mm x_{max} = 91.2 mm
fatigue design for concrete not complied !

additional reinforcement: Δa_{sB1o} = 7.93 cm^2/m Δa_{sB2o} = 3.64 cm^2/m
Δa_{sB1u} = 0.79 cm^2/m Δa_{sB2u} = 0.79 cm^2/m

cross-section data
gross area of concrete: a_c = 30.0 dm^2/m, second moment of area: i_{cs} = 22.5 dm^4/m
moment of resistance: w_{cs} = 15.0 dm^3/m, distance of centre of gravity from upper edge: z_s = 15.0 cm
total area of longitudinal reinforcement: Σ(min a_s) = 26.78 cm^2/m ⇒ ρ_s = 0.89% < 8.00%

material properties for design calculation

concrete	f_{ck}	α	ε_{c2}	ε_{c2u}	n_c	E_{cm}	f_{ctm}
	MN/m^2	-	‰	‰	-	MN/m^2	MN/m^2
C20/25	20.0	0.850	-2.00	-3.50	2.00	29962.0	2.210

design value of compression strength $f_{cd} = \alpha_c f_{ck} / \gamma_c$
strain at reaching the maximum strength ε_{c2}, ult. compr. strain ε_{c2u}
concr. comp. stress $\sigma_c = f_{cd} (1-(1-\varepsilon_c/\varepsilon_{c2})^n)$ for $0 \geq \varepsilon_c > \varepsilon_{c2}$ and $\sigma_c = f_{cd}$ for $\varepsilon_{c2} \geq \varepsilon_c > \varepsilon_{c2u}$
modulus of elasticity E_{cm}, mean value of axial tensil strength f_{ctm}

reinforcem.	f_{yk}	f_{tk}	ε_{su}	E_s
	MN/m^2	MN/m^2	‰	MN/m^2
BSt 500 (A)	500.0	525.0	25.00	200000.0

design yield strength $f_{yd} = f_{yk} / \gamma_s$
design tensile strength $f_{td} = f_{tk} / \gamma_s$
ult. tensile strain ε_{su}, modulus of elasticity E_s

symbols: positive result values marked with -1.0 or **** in tables
refer to incorrect resp. not computable conditions !